Tribute to Margaret Wright on her Election to the National Academy of Sciences

It's a great honor and pleasure for me to give this tribute to Margaret Wright today. I have known Margaret longer than I've known anyone else here; when I met her, I was beginning my graduate study and she was finishing hers. Margaret's field of research is Optimization, and she is one of only a few optimization researchers in the NAS today.

Optimization is a rich and thriving mathematical discipline with an enormous practical impact. Its roots are in the calculus of variations, but its modern history began in 1947 with George Dantzig's Simplex Method for Linear Programming. In fact, Dantzig passed away just a few months ago at the age of 90. He was a member of the NAS and the NAE, and many thought he should have been awarded the Nobel Prize in Economics, which was awarded for Linear Programming in 1975, but went to Kantorovich and Koopmans instead.

A linear program is an optimization problem with a linear objective function and linear **inequality** constraints: the inequalities are what make the problem interesting. The simplex method and its extensions to other problem classes dominated optimization for 40 years, but during the past 20 years a **revolution** has taken place: the **interior point revolution**. Margaret was "in at the beginning" of the interior point revolution, in fact she was in **before** the beginning, as I'll now explain.

Margaret's PhD thesis – which I read from start to finish long ago, and which I still have – proposed a new algorithm for **nonlinear programming**: optimization problems with a nonlinear objective function and nonlinear inequality constraints. Margaret's method, which she called the **barrier trajectory method**, generates a sequence of points that approximates the trajectory of minimizers of a sequence of **log barrier** functions.

The idea of a "log barrier" is to augment the minimization objective with logarithmic terms, specifically the term $-\mu \log c(x)$ for each constraint $c(x) \geq 0$, where μ is a positive parameter. Whatever μ is, the **barrier function** – that is the objective plus the log terms – goes to ∞ as any constraint $c(x)$ goes to 0; thus the log terms constrain the minimizer of the barrier function to the **interior** of the feasible region, where the constraints are positive. As the parameter μ goes to 0, the minimizer of the log barrier function generically converges to a minimizer of the original nonlinear program, normally on the **boundary** of the feasible region.

Margaret was not the first to use log barriers in optimization; the idea goes back to the 50s and Fiacco and McCormick published a book on barrier and penalty methods in 1968 describing a method called SUMT: Sequential Unconstrained Minimization Techniques. However, Margaret **was** the first to devise a method that follows the barrier trajectory **without** solving a sequence of unconstrained problems – problems that are increasingly ill-conditioned. Building on the work of her adviser, Walter Murray, on penalty methods, she showed how one can follow the barrier trajectory by exploiting duality properties and solving a sequence of linear systems. This was fundamentally important work and was completed in 1976.

Then in 1979 and 1984, the optimization world was shaken up twice: first by Khaciyan's announcement that linear programming could be solved in polynomial time by something called the **ellipsoid** method, and then by Karmarkar's **projective** method – also polynomial-time, but far more efficient in practice, and which Karmarkar claimed to be much more efficient than the simplex method. The simplex method has an inherently combinatorial aspect to it as it explores the vertices of the polyhedron of feasible points, although it is very fast in practice. Karmarkar's projective method passes through the interior of this polyhedron. The method received so much publicity and there were so many pictures in the popular press showing how the method passes through the interior instead of going around the outside of the polyhedron that an optimization colleague reported in a paper that his mother had berated him for not having thought of going through the interior years earlier. However, Karmarkar's method did not have any obvious connection with log barriers. It was Margaret and her colleagues at Stanford who soon realized that the projective method was intimately related to log barriers, although no one had thought previously that it would make sense to apply log barriers to **linear** problems. This discovery had a huge impact and led to a lot of subsequent work by many people.

No one uses Karmarkar's projective method any more. The simplex method is still alive and well and used by thousands of people every day, although Dantzig has left us. But it is now generally accepted that the most efficient way to solve really huge linear programs is the primal-dual interior point path-following method – this method approximates what is now called the **central path**, but it is what Margaret called the barrier trajectory. This also seems to be emerging as the most effective way to solve huge **nonlinear** programs.

Margaret has played a major role in the subsequent development of interior point methods and has done lots of other work, which I won't describe except to mention her work on direct search methods, which some of you I'm sure remember from her interview talk nearly 5 years ago. However, I do want to say three other things.

First, Margaret and her coauthors have written two books on optimization, one of which, **Practical Optimization**, is surely one of the most influential books on nonlinear programming ever written. She is well along the way to publishing a third book, which would have appeared by now if we had not diverted her into being chair of computer science.

Secondly, and very appropriately for a chair of computer science, Margaret has had a long commitment to the scientific importance of published software. She and her colleagues at Stanford released many codes, of which one in particular, NPSOL, has remained very widely used even though it is now 20 years old. Margaret was a member of the Numerical Software Working Group of IFIP for 18 years, and was its chair for 4 years.

Lastly, as most of you are well aware, Margaret has a simply incredible record of service to the scientific community. It is probably not too strong to say that her record is second to none, even looking around this room at some of you who have made huge service contributions yourselves. Just to mention a few things, she was President of SIAM, is on the board of trustees at MSRI, has advised NSF and DOE on countless occasions, has served on a delegation to the ICM, is on the Committee for the National Medal of Science, and has served on major review committees in England, Canada and Germany, as well as dozens of other committees. She has also received many honors, ranging from the very first Forsythe award for student contributions to teaching computer science at Stanford to special awards for distinguished public service given separately by the AMS and by SIAM, as well as her election to the NAE and now the NAS. Margaret's dedication, energy, good judgment, personal warmth and amazing connections are legendary. Someone here asked me, when we were recruiting her as chair of computer science, how she would possibly have the time to do the job given all her other activities. I replied that I didn't know, but that if she takes it on, she'll give it the same energy and dedication that she gives everything else. This, as you know, turned out to be the case, although I still don't know how she has done it. Margaret is well known to many as a friend who can be counted on to help out at any time and with little or no notice, assisting with all sorts of things that are far beyond the call of duty. I particularly remember Margaret's past-presidential address to SIAM after her presidential term ended; there was a long standing ovation at the end – thanks from a grateful community.

Margaret, you richly deserve this wonderful honor. Congratulations and thanks for everything!

-Michael Overton, Courant Institute, 26 September 2005