StarScan: A Novel Scan Statistic for Irregularly-Shaped Spatial Clusters

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Detecting Disease Clusters

Location of an informative data stream
- # of ER visits per Zip Code
- # of OTC Drug sales per retailer
- Other novel data sources ...

In the presence of an outbreak, we expect counts of the affected locations to increase.

Effective methods should have high detection power & high spatial accuracy.
Detecting Disease Clusters

Spatial Scan Statistic (Circles) (Kulldorff, 1997)

Clusters locations by regions constrained by shape

High power to detect disease clusters of the corresponding shape

But what about irregular shaped clusters?
Detecting Irregular Disease Clusters

Instead of clustering **ALL locations** within the region together, only the **most anomalous subset of locations** within the region is used.

Increases power to detect irregularly shaped disease clusters.

...but returns **unconstrained subsets** that may not reflect a pattern of interest.

(Neill, 2012)
Sample Data
Sample Data: Circles
Sample Data: Fast Subset Scanning
Sample Data with Noise
Sample Data with Noise: Circles
Sample Data with Noise: Fast Subset Scanning
Sample Data with Noise: Star Scan

We propose a new star-shaped scan statistic ("StarScan") that can more accurately detect smooth irregularly-shaped clusters.
Cross Pattern
Cross Pattern: Fast Subset Scanning
Cross Pattern: Circles
Cross Pattern: StarScan
Real-world examples
Star Scan

- We propose a new technique to detect smooth irregularly shaped clusters.
- Instead of requiring a constant radius (as for circles), StarScan allows the radius to vary, but applies a **penalty** proportional to the total change in radius.
- We propose a new, **dynamic programming**-based solution to find the clusters that maximize the **penalized log-likelihood** ratio statistic.
Expectation-Based Scan Statistics

For location $s_i$:
- Observed: $x_i$
- Expected: $\mu_i$

$H_0: x_i \sim \text{Dist}(\mu_i)$
$H_1(S): x_i \sim \text{Dist}(q\mu_i), q > 1$

$$F(S) = \max_{q>1} \log \frac{P(\text{Data} \mid H_1(S))}{P(\text{Data} \mid H_0)}$$

Large number locations with a moderate risk
Small number of locations with a high risk
Detour: Linear Time Subset Scanning

- Our goal in subset scanning is to find optimal subset $S^*$ such that $F^* = F(S^*)$ where

$$F^* = \max_S F(S) = \max_S \max_{q>1} \log \frac{P(Data|H_1(S))}{P(Data|H_0)}$$

- In order to find best subset we need to evaluate exponential number of subsets.

For most (exponential family distributions) of the scoring functions, we need to evaluate only linear number of subsets.
Additive Linear Time Subset Scanning

\[ F(S) = \max_{q>1} \log \frac{P(Data \mid H_1(S))}{P(Data \mid H_0)} \]

\[ H_0 : x_i \sim \text{Dist}(\mu_i) \]

\[ H_1(S) : x_i \sim \text{Dist}(q\mu_i) \quad q > 1 \]

Conditioning ALTSS functions on the relative risk, \( q \), allows the function to be written as an **additive** set function over the data elements \( s_i \) contained in \( S \).

**Poisson example:**

\[ F(S) = \max_{q>1} F(S \mid q) \]

\[ F(S \mid q) = \sum_{s_i \in S} x_i (\log q) + \mu_i (1 - q) \]
Conditioning on relative risk

By conditioning on relative risk \((q)\) each element is either “positive” or “negative”

This simplifies the maximization over subsets

- Include only the points whose contribution to LLR are positive while minimizing change in radius

\[
F(S) = \max_{q > 1} \sum_{s_i \in S} \left[ x_i (\log q) + \mu_i (1 - q) \right]
\]
Star Scan: Fundamentals

- The score of subset \((S)\) is dependent on the following four characteristics:
  - Cumulative sum of observed: \(\sum x_i = X(S)\)
  - Cumulative sum of expected: \(\sum \mu_i = \mu(S)\)
  - Total change in radius to form a subset: \(R(S)\)

- We propose a dynamic programming based solution to find optimal subset that maximizes the score of subset \((S)\)

\[
F_{\text{starscan}}(S \mid q) = F(S \mid q) - \lambda \ast R(S)
\]
Dynamic Programming for Star Scan
Dynamic Programming for Star Scan

Steps Ahead

- Best of
  - Constant Radius
  - Best path via 1, 2, 3, 4, 5

Start Location

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$X(S), \mu(S)$

$\Delta(S), R(S)$
Star Scan generalizes FSS and Circles

- The penalty parameter can be used to generalize Star Scan

\[ F_{starscan}(S \mid q) = F(S \mid q) - \lambda \times R(S) \]

- \( \lambda \) is the penalization parameter
  - High value of \( \lambda \): Circles (Kulldorff, 1997)
  - Low value of \( \lambda \): Fast Subset Scan (Neill, 2012)
Star Scan: Challenges

- Dynamic programming is easy for a given relative risk \( q \) as each element is either “positive” or “negative”, that is,

\[
F_{\text{starscan}}(S \mid q) = F(S \mid q) - \lambda \times R(S)
\]

\[
F^*(q) = \max_S F_{\text{starscan}}(S \mid q)
\]

- However the optimal score \( F^* \) is given by

\[
F^* = \max_{q>1} \max_S F_{\text{starscan}}(S \mid q)
\]
DP for Star Scan: Solutions

- We can either grid search for the values of $q$ in the range of possible values.

- Or use branch and bound technique in order to find the optimal value of $q$. 
Bayesian Aerosol Release Detector (BARD)

Hogan et al; 2007

Simulates anthrax spores released over a city

Two models drive the simulator:

<table>
<thead>
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<th>Dispersion</th>
<th>Infection</th>
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<td>Which areas will be affected?</td>
<td>How many infected people in an area?</td>
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<tr>
<td>Weather data</td>
<td>Demographic data</td>
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<tr>
<td>Gaussian plumes</td>
<td>Increased ER visits with respiratory complaints</td>
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</table>
Results: Spatial Overlap

\[ \text{Overlap} = \frac{A \cap B}{A \cup B} = \]

\[ \text{Overlap} = 1 \quad \text{Perfect Match} \]

\[ \text{Overlap} = 0 \quad \text{Completely Disjoint} \]
BARD Results: Spatial Overlap

Overlap vs Neighborhood Size for Circles, FSS, and StarScan.
BARD Results: Time to Detect at a fixed false positive rate

Time to Detect in Days

Neighborhood Size

- Circles
- FSS
- StarScan
Simulated Injects in real-world Emergency Department data.
Simulated Injects (continued)

**Overlap**

- StarScan
- FSS
- Circles

**Time to Detect in Days**

- StarScan
- FSS
- Circles
Conclusion

- We propose StarScan to find irregularly-shaped clusters more accurately than either the circular scan or unconstrained fast subset scan.

- StarScan was compared with circular scan and fast localized subset scan on simulated respiratory outbreaks and bioterrorist anthrax attacks injected into real-world Emergency Department data.

- Given a small amount of labeled training data, StarScan learns appropriate penalties for both compact and elongated clusters, resulting in improved detection performance.
Thank Q?