Mehryar Mohri
Advanced Machine Learning 2024
Courant Institute of Mathematical Sciences
Homework assignment 2
April 09, 2024
Due: April 23, 2024

1 Swap regret for large expert spaces

Leverage the results presented in class to give a swap regret algorithm tailored for large expert spaces. You should give a full description of your algorithm and provide a detailed proof of the corresponding regret guarantee.

2 Mirror Descent with smooth functions

Consider the Mirror Descent algorithm in the *offline setting*, that is $f_t = f$ for all t. Assume that f is β-smooth. We will adopt the notation used in the online convex optimization lecture.

1. Prove that the following inequality holds for any $t \geq 1$:

$$\langle \nabla \Phi(\mathbf{w}_{t+1}) - \nabla \Phi(\mathbf{v}_{t+1}), \mathbf{w}_{t+1} - \mathbf{w}^* \rangle \leq 0.$$

2. Use that inequality to show that for any $t \geq 1$ the following inequality holds:

$$\delta f(\mathbf{w}_t) \cdot (\mathbf{w}_{t+1} - \mathbf{w}^*) \leq \frac{1}{\eta} [B(\mathbf{w}^* \parallel \mathbf{w}_t) - B(\mathbf{w}^* \parallel \mathbf{w}_{t+1}) - B(\mathbf{w}_{t+1} \parallel \mathbf{w}_t)].$$

3. Choose $\eta = \frac{\alpha}{\beta}$. Show that MD with this choice of the learning rate and α-stongly convex mirror map converges in $O(1/T)$. Give an explicit upper bound in terms of α, β, and $D^2 \geq B(\mathbf{w}^* \parallel \mathbf{w}_1)$.