Some Discussion of Static Gripping and Its Stability

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Abstract

Recently, there has been a large amount of research in dextrous manipulation. The main issue has been how to control mechanical hands so that they can perform manipulation tasks with the same dexterity and sensitivity as the human hands. In order to achieve sophisticated algorithms for grasping, compliance control and manipulation, the nature of the contact wrenches, twists and compliance of the fingers have to be well-understood. This paper concentrates mainly on one area of dextrous manipulation: static grasping. The two main approaches to the problems of grasping are those motivated by the study of human hands and those motivated by the physical and mechanical properties of the grasps. This includes studies of contact types, number of fingers required to achieve grasps, equilibrium, stability, and compliance.

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1 Introduction

Dextrous Manipulation means handling objects skillfully and adroitly and involves operating manipulators with actively powered fingers. We would like to think of a robot as a machine which can be instructed to perform at least any task which a human can, at least as well as a human; however, the current state of robotics research is far from this goal. At this point, although there exist reasonably accurate mechanical manipulators, we do not know how to use them effectively, and therefore they are useful only in simple application domains, such as welding and spray painting.

A dextrous hand is one with multiple articulated fingers (i.e. fingers with two or more powered joints), each of which is an open kinematic chain, and possibly a palmar surface. It contains a large number of degrees of freedom, and can achieve fine resolution with high accuracy and at high speeds. Each of the fingers should contain position and force sensors and may additionally contain tactile sensors.

The first question to be answered is "Why use dextrous hands instead of just conventional grippers?" Hands have a number of advantages over conventional grippers. [2] Hands provide versatility for fine motions, and hence are less error prone. They also allow control of gripping forces which provide more efficient grasps. This ranges from securely grasping an object in order to prevent slipping to grasping a delicate object so that it is not damaged. In addition, a multi-fingered hand can grasp unusually shaped objects which a parallel gripper cannot. And, unlike simply a set of robot arms working cooperatively, a hand may have a palm, there may be coupling between the fingers as there is in the human hand, and since there is only one arm, manipulation is not as complex.

The science of manipulation, which is only a subfield of robotics, is a broad field in itself, comprising of many topics such as gripper design, grasping, control, tactile sensing, and task planning. The main problem is determining how to use the fingers of the hand not only to obtain an effective grasp of an object (which may be of unknown shape and orientation), but also to manipulate the object in order to perform a given task. In a broad sense, the problems involved include [21]:

- *Control Issues*: Processing and utilization of force/torque and spatial informations necessary in geometric-, positional-, and force-feedbacks while correcting for errors which may occur.
- *Geometric Issues*: Adapting to bodies of known and unknown shape, especially, in the absence of sophisticated sensory data.
- *Integration Issues*: Managing information about the force, torque, mobility and compliance available in the system, such as a priori knowledge and information obtained from sensors.
- *Task Planning Issues*: Devising appropriate trajectories.

In order to intelligently deal with each of these issues the nature of the contact wrenches, twists (mobilities), and compliance of the fingers must be well-understood in each of the subtasks of dexterous manipulation. Basically, there are four subproblems: *grasping*, *homogeneous manipulation*, *high-level task planning*, and *design of hands and tools for manipulation*.

The grasping issues are

- Static issues: How do the contact types restrict the nature of the force? What restrictions are caused by the contact surface, i.e. What effects do unreachable areas of the object have on the grasp? How are the available degrees of freedom dealt with? How does the grasp depend on the geometry of the hand and the object?

- Existence of a grip. This depends on the properties of the object and the characteristics of the hand.

- Characterization of a grip. This is the measure of "goodness" of the grip. Can the desired task be performed efficiently using this grip?

- Synthesis of grips. Devising efficient algorithms to determine the contact points and forces.

Homogeneous manipulation characterizes control tasks at the small-scale, where the parameters vary continuously. They include many simple tasks such as turning a knob and avoiding collisions while manipulating an object. The main issues are

- Selection of a set of object-finger transformations that will allow object force/position targets to be mapped onto finger force/position targets. If gripping points do not change the transformation would be constant and linear. However, in general, the gripping points will change (even without changing the grips significantly) and the transformation will be rather complex (e.g. nonlinear and time-varying).

- Control of the joint actuators (dc motors, pneumatic pistons, etc.) to achieve the desired finger targets and to correct for modeling errors. How can the large number of actuators be managed? Which kind of sensory data can be fed back into the servo-loops and how would it be used?

- Specification of a (single) control law that would cause the gripped object to behave locally in a prescribed way (e.g. remote center of compliance, stiffness control).

High-level task planning includes

- Developing a sequence of homogeneous manipulations to execute the task while providing the necessary transition procedures (which might not be quasi-static).

- Transition of grips. Most tasks require a number of different grips. In order to perform these tasks, the robot must be able to change from one grip to another, possibly very different, grip. This change must be done in real time without losing control of the object.

Design of hands and tools for manipulation includes

- Deciding what kind of hands and tools to build. Do we want anthropomorphic hands or a possibly more efficient hand design, i.e. hands designed to execute specific tasks more efficiently? Do we need to redesign tools for these hands?

This paper does not attempt to deal with all of these problems; rather, it discusses the some of the issues involved in static grasping. In attempting to solve these problems, researchers have taken two basic approaches; one is a physiological approach motivated by the study of human hands and the other is motivated by the physical and mechanical properties of hands and objects.

The human hand is a versatile end effector which can perform intricate tasks. Studying human grasps can lead to a grasp taxonomy which can assist robot hands in choosing an appropriate grasp to perform a specific task, and can also be used to design efficient grippers.

Another approach involves studying the physical and mechanical properties of grasps and understanding how the actions of the robot affect the motions of the object. For instance, when a finger touches an object, will it slide away? Will it settle into a secure grasp? Will it settle into a grasp which allows motion relative to the manipulator? How do we model the fingers? What forces are transmitted by these "fingers"? How compliant are the fingers? Is there friction?

It should be noted that these two approaches are not competing. To some extent, they deal with different aspects of the same problem, and therefore they complement each other. Whereas the mechanical approach seems more suitable for low level tasks (homogeneous manipulations), the physiological approach is likely to be more suitable for high level tasks.

This paper is divided into two main parts, the first of which deals with studies on human grasping and grasp-taxonomies, while the second part deals with physical and mechanical issues such as contact types, number of fingers required to achieve grasps, equilibrium, stability, and compliance. The final section summarizes the results.

The mathematical convention used throughout this paper is as follows: scalars are denoted by lowercase italics, vectors are denoted by lowercase boldface, and represented by a row-vector, and matrices are denoted by script uppercase.

Part I
Physiological Approach

2 Grasping Taxonomies

In an effort to simplify the choice of grasp a robot has to make, numerous attempts have been made to categorize grips by studying grasps of the human hand. The human hand is capable of a wide range of grasps, and the grasp selection depends on the size and shape of the object as well as the task to be performed. Studying how these factors influence the grip choice leads to a taxonomy which provides a systematic way to choose an appropriate grasp for a particular set of task requirements and object characteristics.

2.1 Medical Literature

Human grasps were first categorized in medical literature by Schlesinger [20] and are summarized in Taylor and Schwartz [23]. This classification is based on the shape of the object rather than the task to be performed. The six types of grasps are the cylindrical grasp, the tip grasp, the hook grasp, the palmar grasp, the spherical grasp and the lateral grasp. (fig. 1)

Napier [16] argues that when one grasps an object for whatever purpose, the major concern is the stability of the grasp. He therefore bases his classification on *how* stability is achieved, which

Figure 1: Six basic grasps as defined by Schlesinger

leads to two major types of grips:

1. *Power Grip*: The object is held in a clamp formed by the partly flexed fingers and the palm, while counter pressure is applied by the thumb which lies more or less in the plane of the palm.

2. *Precision Grip*: The object is pinched between the flexor aspects of the fingers and the opposing thumb.

He then goes on to discuss the factors which influence the posture of the hand.

The first factor is the shape of object, which for many medical researchers, such as Taylor and Schwartz, is the main factor to consider when determining the grip. In contrast to them, Napier claims that in fact, the shape of the object has no bearing on the posture adopted by the hand, since most objects may be held in a number of grips with equal security, and that the grip depends solely upon the purpose to which the object is to be put.

The next factor discussed is the size of the object. For medium size objects, both grips will provide stability. For larger sizes, however, the grip chosen is the one which provides the greatest span of the hand without compromising stability; this is most likely a power grip. This factor has no general applications.

Additional factors which may sometimes influence type of grip employed are weight, texture, temperature, and wetness of object.

The most important influence is the intended activity. During the performance of a purposive prehensile action, the posture of the hand bears a constant relationship to the nature of that activity. If prehensile activities are to be regarded as the application of a system of forces in a given direction, then the nature of the prehensile activity can be resolved into two concepts: *precision* and *power*. (The two, however, are not mutually exclusive; for some tasks, both may be required.) Finally, the pattern of the chosen grip is based upon the nature of the task.

2.2 Grasping in a Manufacturing Environment

Cutkosky and Wright [3] approached the problem in order to design efficient robot grippers for a manufacturing environment. They carried out a series of protocol analysis experiments with machinists and classified these and other machining grips. These have led to the taxonomy of human grasps which they organized as a hierarchical tree. They extended Napier's basic categories by considering the task and geometric considerations that influence the pattern of the grip. (fig. 2)

Moving from left to right along the tree, the grasps become less powerful yet more dextrous and the grasped objects become smaller. The wrap grips are the most powerful yet least dextrous and therefore require that all manipulation must be done with the wrist. By contrast, the precision grips are much more delicate. Moving from top to bottom, the choices range from task-directed grasps to object-directed grasps. As with any hierarchical classifications, there are some exceptions to these rules.

Often, a grip must be continuously modified in order to retain maximum efficiency in relation to the objective sought in the grip. In these cases, the influence of forces and torques on the choice of grip is most apparent. For example, in unscrewing a knob, the grip changes from grip 11 to grip 13. When similar tasks are performed with slightly different tools, other factors come into play. For example, the grip may shift from 1 to 5 for a lighter tool, or from 13 to 14 for a very small object.

2.3 Virtual Fingers

Iberall's [7] classification is based on the fact that in practice, the human hand uses a combination of grasps. Most of the other taxonomies fail due to their inability to explain all the exceptions which occur as a result of task constraints. A classification is required which describes a prehensile posture in goal-directed terms. It should be able to describe task constraints and hand postures in terms of the number of, directions, and strength of the forces to be applied. In order to select a grasp, specific characteristics must be identified with each domain of interaction. Once the grasp has been chosen, the object characteristics are used to particularize the grasp components for the object. The same grasp may use different physical fingers depending on the object characteristics. For example, the same grasp is used to grasp mugs of different size even though a different number of fingers may be used. Therefore, the grasp is defined in terms of *virtual fingers*, and is later mapped onto the physical fingers. The concept of virtual fingers provides independence from physical hand structure and particular object models.

According to Iberall, there are three basic methods the hand can employ to apply opposing forces around an object for a given task, each of which use two VF's (virtual fingers) (fig. 3):

1. *Pad Opposition* is between the thumb pad (VF1) and the finger pads (VF2). This grip provides great flexibility in finely controlled motions, but little stability.

Figure 2: Cutkosky and Wright's Taxonomy of Human Grasps

Pad Opposition Palm Opposition Side Opposition

Figure 3: Iberall's Basic Oppositions that make up human prehension

2. *Palm Opposition* is between the palm (VF1) and the digits (VF2). It sacrifices flexibility in favor of stability.

3. *Side Opposition* is either between the thumb pad (VF1) and the side of the index finger (VF2) or between the sides of the fingers. This grip is a compromise between flexibility and stability.

Lyons [10, 11] used the concept of virtual fingers to classify grips as part of a computational model which he built, since in order to build a gripping device which can approach the versatility of the human hand, a high level control structure is needed. He studied the human hand and made some observations. The hand preshapes for the grasp while it reaches for it. While the reach is concerned with the object's position, the grasp is concerned with the object characteristics as well as the intended object usage.

He therefore associates with each grasp a preshape finger configuration for that grasp and the degrees of freedom of the hand once the object has been grasped. He classifies grasps into three major categories and calls this classification SSG, a Simple Set of Grasps. As in Iberall's classification, there is a power grasp, a precision grasp, and a grasp which provides some power and some precision. (fig. 4)

1. *Encompass grasp.* This is a power grasp; it provides no manipulative ability but secure affixment. The hand completely envelopes the object. Manipulation can only be accomplished through the arm and wrist and this is used for basic 'pick-and-place' tasks.

2. *Lateral grasp.* This is basically the pinch grasp. The flat surfaces of the fingers are used to grip the object. It provides some manipulative ability and some stability.

3. *Precision grasp.* The object is held between fingertips. It allows arbitrary motion but provides little stability.

Each of these grasps must be particularized to suit the size and shape of the target object. Therefore, Lyons derives a table relating functionalities and object characteristics to a given grasp; he calls this the *grasp index*. The functional input to grasp selection is *firmness* of the

Encompass Grasp Lateral Grasp Precision Grasp

Figure 4: Lyons' grasps

grasp required and *precision* of the grasp required. This results in four categories: no precision and no firmness, no precision and firmness, precision and no firmness, precision and firmness. Two object characteristics are considered: *size*, which can be small, long, or large, and *shape*, which is either flat or round. Thus the grasp index tells how required functionality and object characteristics are used to find a suitable stereotype grasp.

Lyons divides grasping into three phases:

1. *Preshape*: This is where the hand is configured in preparation to grip the object while it is reaching for the object.

2. *Acquisition* or *Gripping*: As the fingers move to contact the object, they have low stiffness, and once the contacts are in place, the contact forces required to secure the object are calculated. There are three cases:

   (a) *Simple Acquisition*: The hand moves toward the object in the preshape configuration and grasps it.

   (b) *Two-phase Acquisition*: The object is acquired in a temporary grasp and is then manipulated to the final configuration.

   (c) *Constrained Acquisition*: It is impossible for the hand to approach the object with the chosen grasp and preshape. Therefore either the grasp or preshape must be modified.

3. *Manipulation*: A table is used to determine how to move the object based on the degrees of freedom left in the hand. This ranges from moving only the wrist to the hand being able to impart arbitrary forces and moments to the object.

2.4 Comparison of Taxonomies

Although there are many different classifications, the underlying theme of *precision vs. stability* as introduced by Napier, influences all the taxonomies. This shows the importance of precise and stable handling in performing dextrous tasks.

Cutkosky and Wright present a good and thorough classification for a machine environment. The hierarchical representation is neat and clear. However, there are overlaps and exceptions. One example they mention is that a spherical grasp may be more or less dextrous than a medium wrap, depending on the size of the sphere. Since in a restricted machine environment, the characteristics of the objects to be manipulated is known a priori, this does not present a problem. However, if we wish to expand this taxonomy to other domains which are more complex and less constrained, a tree representation may not be accurate, yet nevertheless it is probably the best and clearest approach.

Lyons, like Cutkosky and Wright, bases his classification on *precision vs. stability*. Rather than expanding into a tree, for each of his primary grips, he considers different characteristics, from which results a table (the grasp index). However, his grasp index is simple and limited. It considers only coarse descriptions of characteristics in some unknown restricted environment. An attempt to enlarge this table for a general environment would probably render the table unwieldy and overly complex.

Iberall attempted to simplify the taxonomy by showing that every grip is really two virtual fingers acting along some pad of opposition. She mapped the grips from the other taxonomies into her own. She criticized the previous taxonomies for not explaining exceptions due to task constraints. While her criticisms are valid, her new taxonomy does not address these issues adequately. Furthermore, more than any of the other classifications, her taxonomy is more dependent on the characteristics of the human hand, and may not be very useful for many non-anthropomorphic grippers. It is not apparent how this taxonomy would help simplify the grasp choice for a robot task; presumably she uses a table like Lyons.

All of these taxonomies functioned only in limited domains. Cutkosky and Wright did their study in a manufacturing environment. Lyons attempted to be more general, but when he was forced to particularize the grasp, he could only handle a limited number of cases in his grasp index.

At this point we must ask ourselves, are we going to use these taxonomies to design grippers or to assist in selecting a grasp? What kind of environment are we dealing with? Is there another way to quantify these grasps, perhaps mathematically? We can study how humans perform tasks in particular domains or see if there are better ways to perform grasps, and design hands to perform those domain specific tasks efficiently, or we can use these taxonomies and try to better understand how to particularize a grasp in order to perform a specific task and how to achieve these precise and stable grasps.

Part II

Mechanical and Physical Approach

While much research has been done in categorizing grasps to assist in grasp selection, another approach to grasp selection has been to study the physical and mechanical properties of the hand, the object, and the contact between them. There have been quite a number of different grasps that are discussed in the literature. Although most people might not think before grasping certain objects, there are many factors that influence the choice of a grip. Researchers have considered different factors, such as contact types, friction, compliance, and stability, as well as the intended task, in their analyses. Due to the numerous factors involved, the problem of determining what grip a robot should make is a complex one, and evaluating a grip is difficult as well.

This part discusses some of these issues. Section 3 discusses different contact models and section 4 discusses different grasps in physical terms.

3 Contacts

Before we can study how to find a grip and determine its properties, we must first consider the interaction between the fingertips and the object being grasped. It is therefore important to study the mechanics of constraint and freedom that take effect when two bodies come into contact. The contact causes forces and moments to be transmitted to the objects, and consequently, the relative motion between the two bodies is determined by the geometric and friction properties of the contact.

3.1 Contact Types Based on Screw Theory

Salisbury's [14] analysis is based on screw theory which uses six parameters, so there are six degrees of freedom (relative motion), and hence six classes of contacts (excluding the trivial case of no contact). A surface can experience three types of contact: point contact, line contact, and plane contact. By studying how the motion of a body is constrained by the various types of contact and considering the effect of friction, he derives the contact types shown in the table below. The constraints imposed by each type of contact are described in terms of the degrees-of-freedom allowed and in terms of screw systems of motion and force associated with each contact type.

CONTACT TYPE	REMAINING DEGREES OF FREEDOM
No Contact	6
Frictionless Point Contact	5
Frictionless Line Contact	4
Friction Point Contact	3
Frictionless Plane Contact	3
Soft Finger	2
Friction Line Contact	1
Planar Contact with friction	0

This formulation is neat and makes the calculations simple, however it is not always accurate for certain kinds of contact. For instance, it does not take into account surface deformation under load, which, in addition to friction and surface geometry, also effects the strength and stability of a grip.

pointed curved flat very soft soft, curved

Figure 5: Cutkosky's contact types

3.2 Contact Types Based on Fingertip Geometry

Cutkosky [2] defines a different set of models. Rather than studying just contacts between rigid bodies, he considers different types of fingertips and defines a set of models that may be used to characterize the fingertip geometry: *point contacts*, *hard curved contacts*, *flat contacts*, *soft curved contacts*, and *very soft contacts*. (fig. 5)

Cutkosky compared the different kinds of fingertips. He analyzed the forces and torques they exert, as well as the effects of friction and the sizes of the radii of curvature of the fingertips. He found that the rolling of curved fingers causes the contact area to shift with respect to the object. As the radius of curvature of a curved finger approaches zero, the contact behaves like a point contact with friction, and as the radius of curvature becomes very large, the contact approaches a planar contact with friction.

Since fingers deform, Cutkosky studied deformation of fingers and how it affects contacts. The fingertip compliance is considered in addition to the finger joint compliance. For a small contact area, the fingertip becomes more compliant with respect to rotations and approaches the point contact model. For larger contact areas, the translational compliance is larger, and the contact approaches a planar contact with friction.

Human fingertips exhibit rolling and deformation. The model of the human fingertip is soft and curved, and does combine the attributes of the soft and the curved fingertips.

Cutkosky then studied the major differences between the contact types. He found that for a pointed fingertip, the problem was two-dimensional when only rotations were considered. For curved fingertips, the grasps proved to be more stable on account of the rolling of the fingertips. He also discovered that there is an important relationship between the radii of curvature of the fingertips and the object being grasped. The quantification of the size of the radius of curvature of the fingertip depends on the size of the object, so that for large objects, it is reasonable to consider the point contact model.

Salisbury provides a simple classification of contact types, while Cutkosky does a more in-depth study of contacts and attempts more realistic models of contacts. However, we shall see in the coming sections, that most analyses consider the point-contact with friction, since it is simplest to deal with mathematically, although quite a few consider the soft finger model which models a human finger.

4 Grasping: Terminology and Theory

In an effort to bring several issues in this area into sharp focus, we shall make the following simplifying assumptions: Consider an idealized robot hand, consisting of several independently movable force-sensing fingers; this hand is used to grasp a rigid object B. Furthermore,

- (*Smooth Body*) B is a full-bodied (i.e. no internal holes) compact subset of the Euclidean 3-space. Furthermore, B has a piece-wise smooth boundary ∂B. We remark that all the discussion here applies to the case where B is in the Euclidean 2-space as well, but we will forego explicit treatment of this case, for simplicity.

- (*Point Contact*) For each finger-contact on the body, we may associate a nominal point of contact, $\mathbf{p} \in \partial B$. We let ∂B^* denote the set of points $\mathbf{p} \in \partial B$ such that the direction $\mathbf{n}(\mathbf{p})$ normal to ∂B at $\mathbf{p}$ is well-defined; by convention, we pick $\mathbf{n}(\mathbf{p})$ to be the unit normal pointing into the interior of B.

  For each such point $\mathbf{p}$, we can define a wrench system $\{\Gamma^{(1)}(\mathbf{p}),\ \Gamma^{(2)}(\mathbf{p}),\ \ldots,\ \Gamma^{(m)}(\mathbf{p})\}$, $(0 \leq m \leq 6)$, where the number and screw-axes of the wrench system depend on the contact type. Some of these wrenches can be *bisense* (i.e. can act in either sense) and the remaining wrenches, *unisense*. In a similar manner, we can also associate a twist system with each nominal point of contact. (For a discussion of screw theory, and in particular, wrenches and twists, see [6] and [19].)

- (*Compliance*) We will consider two cases: When the fingers are stiff—the force/torques applied at the fingers are generated by some actuators whose mechanics need not concern us, and when the fingers are compliant—the contacts are modeled as a set of independent *linear* or *angular* springs and the force/torques are derived from the potential energy (a scalar quantity). Thus, in the second case the wrenches are *monogenic*, while in the first case the wrenches may be *polygenic*.

Many interesting special cases occur, depending on how we model the *static friction* and the *stiction* between the fingers and the body B. In the case, where the contacts are frictionless, a finger can only apply force $\mathbf{f}$ on the body in the direction $\mathbf{n}(\mathbf{p})$ at the point $\mathbf{p}$. Also if the fingers are non-sticky, then the force $\mathbf{f}$ has a non-negative magnitude, $f = \mathbf{f} \cdot \mathbf{n}(\mathbf{p}) \geq 0$. Such grips are also known as 'positive grips'. The wrench system associated with each point is:

$$\Gamma(\mathbf{p}) = \{[\mathbf{n}(\mathbf{p}), \mathbf{p} \times \mathbf{n}(\mathbf{p})]\}$$

We first consider the case in which the finger forces are not necessarily monogenic. Corresponding to a set of finger-contacts, we have a system of n wrenches,

$$\{\mathbf{w}_1, \ldots, \mathbf{w}_k, \mathbf{w}_{k+1}, \ldots, \mathbf{w}_n\},$$

the first k of which are bisense and the remaining last $n-k$ of the wrenches are unisense. Let us assume that the magnitudes of these wrenches are given by the scalars f_i's

$$\{f_1, \ldots, f_k, f_{k+1}, \ldots, f_n\},$$

where $f_1, \ldots, f_k \in \mathbb{R}$ and $f_{k+1}, \ldots, f_n \in \mathbb{R}_+$, and not all the magnitudes are zero. We call such a system of wrenches and the wrench-magnitudes, a *grip*, G, and say that this grip G generates an external wrench $\mathbf{w} = [F_x, F_y, F_z, \tau_x, \tau_y, \tau_z] \in \mathbb{R}^6$, if

$$\mathbf{w} = \sum_{i=1}^{n} f_i \, \mathbf{w}_i,$$

or in a matrix form

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^T & \mathbf{w}_2^T & \cdots & \mathbf{w}_n^T \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$$

that is,

$$\mathbf{w}^T = \mathcal{W}\mathbf{f}^T,$$

where $\mathcal{W}$ is a $6 \times n$ matrix, as given above. The vector $\mathbf{f} \in \mathbb{R}^k \times \mathbb{R}_+^{n-k}$. If $\mathbf{f}$ is a solution to the above equations, then it can be written as $\mathbf{f} = \mathbf{f}_p + \mathbf{f}_h$, where $\mathbf{f}_p \in \mathbb{R}^n$ is a particular solution (assumed to be orthogonal to the NullSpace($\mathcal{W}$) and thus unique) and $\mathbf{f}_h$ is a homogeneous solution in NullSpace($\mathcal{W}$) for the wrench equation. Furthermore, if $\mathbf{f}_h^{(1)}, \ldots, \mathbf{f}_h^{(q)}$ ($q \geq n - 6$) is a fixed orthonormal (linear) basis for NullSpace($\mathcal{W}$) then the solution

$$\begin{aligned} \mathbf{f} &= \mathbf{f}_p + \mathbf{f}_h \\ &= \mathbf{f}_p + \lambda_1 \, \mathbf{f}_h^{(1)} + \cdots + \lambda_q \, \mathbf{f}_h^{(q)} \\ &\qquad \text{where } \lambda_i \in \mathbb{R}, \quad i = 1, \ldots, q. \end{aligned}$$

Salisbury and others ([14]) term $\lambda_i = \mathbf{f}_h^{(i)} \cdot \mathbf{f}$ as the i^{th} *internal force*. However, these values are strongly dependent on the choice of the basis for the nullspace of $\mathcal{W}$, and not all values of internal forces will result in a feasible set of finger force targets.

If the external force generated is $\mathbf{0}$ then we call G an *equilibrium grip*, in which case the following sum

$$\sum_{i=1}^{n} f_i \, \mathbf{w}_i$$

vanishes. While synthesizing grasps, we need to ask the following question: Does the system of equations, given by

$$\sum_{i=1}^{n} f_i \, \mathbf{w}_i = \mathbf{0}, \quad \text{where } [f_1, \ldots, f_n] \in \mathbb{R}^k \times \mathbb{R}_+^{n-k},$$

have a nontrivial solution? The answer is true if and only if one of the following conditions holds:

1. $\mathbf{w}_1, \ldots, \mathbf{w}_k$ are linearly dependent,

2. The origin lies in the convex hull of $\mathbf{w}_{k+1}$, ..., $\mathbf{w}_n$:

$$\mathbf{0} \in \text{conv } (\mathbf{w}_{k+1}, \ldots, \mathbf{w}_n), \quad \text{or}$$

3. The intersection of the linear hull of $\mathbf{w}_1$, ..., $\mathbf{w}_k$ and the convex hull of $\mathbf{w}_{k+1}$, ..., $\mathbf{w}_n$ is nonempty,

$$\text{lin } (\mathbf{w}_1, \ldots, \mathbf{w}_k) \cap \text{conv } (\mathbf{w}_{k+1}, \ldots, \mathbf{w}_n) \neq \emptyset.$$

In particular, if $k = 0$, as in a positive grip, then the condition reduces to the following:

$$\mathbf{0} \in \text{conv } (\mathbf{w}_1, \ldots, \mathbf{w}_n).$$

This brings us to the concept of a *closure grasp*. A system of wrenches $\mathbf{w}_1$, ..., $\mathbf{w}_n$ (as before) is said to constitute a *force/torque closure grasp*[1] if and only if any arbitrary external wrench can be generated by varying the magnitudes of the wrenches (subject to the constraints imposed by the senses of the wrenches). A necessary and sufficient condition for a closure grasp is that the (module) sum of the linear space spanned by the vectors $\mathbf{w}_1$, ..., $\mathbf{w}_k$ and the positive space spanned by the vectors $\mathbf{w}_{k+1}$, ..., $\mathbf{w}_n$ is the entire $\mathbb{R}^6$:

$$\text{lin } (\mathbf{w}_1, \ldots, \mathbf{w}_k) + \text{pos } (\mathbf{w}_{k+1}, \ldots, \mathbf{w}_n) = \mathbb{R}^6.$$

Let us denote, by L, the linear space lin $(\mathbf{w}_1, \ldots, \mathbf{w}_k)$, and, by $L^\perp$, the orthogonal complement of L in $\mathbb{R}^6$. Let π be the linear projection function of $\mathbb{R}^6$ onto $L^\perp$ whose kernel is L. Then by a lemma due to Bonnice and Klee (Lemma 2.7. in [1]) we see that a necessary and sufficient condition for a closure grasp is

$$\text{lin } (\mathbf{w}_1, \ldots, \mathbf{w}_k) + \text{pos } (\mathbf{w}_{k+1}, \ldots, \mathbf{w}_n) = \text{lin } (\mathbf{w}_1, \ldots, \mathbf{w}_k) + \text{pos } (\pi\mathbf{w}_{k+1}, \ldots, \pi\mathbf{w}_n) = \mathbb{R}^6.$$

The above equation in turn is equivalent to the following conditions:

$$\text{pos } (\pi\mathbf{w}_{k+1}, \ldots, \pi\mathbf{w}_n) = L^\perp$$

or

$$\mathbf{0} \in \text{int conv } (\pi\mathbf{w}_{k+1}, \ldots, \pi\mathbf{w}_n)$$

in $L^\perp$. Again, if $k = 0$ then the above condition reduces to the following:

$$\mathbf{0} \in \text{int conv } (\mathbf{w}_1, \ldots, \mathbf{w}_n).$$

Let us assume that $\dim(L) = d$. Then there is a linear basis W of L

$$W = \{\mathbf{w}_{j_1}, \ldots, \mathbf{w}_{j_d}\} \subseteq \{\mathbf{w}_1, \ldots, \mathbf{w}_k\}$$

which when adjoined with a set of vectors W',

$$W' = \{\mathbf{w}_{j_{d+1}}, \ldots, \mathbf{w}_{j_6}\} \subseteq \{\mathbf{w}_{k+1}, \ldots, \mathbf{w}_n\}$$

[1]Our definition of *force/torque closure* is somewhat non-standard.

yields $\widehat{W} = W \cup W'$, a linear basis of $\mathbb{R}^6$. Thus under the condition that we have a closure grasp, we can find $\mathbf{g} \in \mathbb{R}^k \times \mathbb{R}_+^{n-k}$ such that

$$-\left(\mathbf{w}_{j_{d+1}} + \cdots + \mathbf{w}_{j_6}\right) = \sum_{i=1}^{n} g_i\ \mathbf{w}_i,$$

and thus

$$\sum_{i=1}^{n} f_{h,i}\ \mathbf{w}_i, \quad = \quad \mathbf{0}, \quad \text{where } \mathbf{f}_h \in \mathbb{R}^k \times \mathbb{R}_+^{n-k}, \text{ and } f_{h,j_{d+1}} > 0, \ldots, f_{h,j_6} > 0.$$

Now any external wrench $\mathbf{w}$ can be expressed as a linear combination of the vectors in the basis $\widehat{W}$. Thus there is a vector $\mathbf{f}_p \in \mathbb{R}^n$, whose non-zero entries are in the positions $j_1, \ldots, j_6$, and

$$\mathbf{w} = \sum_{k=1}^{6} f_{p,j_k}\ \mathbf{w}_{j_k} = \sum_{i=1}^{n} f_{p,i}\ \mathbf{w}_i.$$

Since $\mathbf{f}_p$ is a particular solution and $\mathbf{f}_h$ is a homogeneous solution to the equation:

$$\mathbf{w}^T = \mathcal{W}\ \mathbf{f}^T,$$

we see that $\mathbf{f} = \mathbf{f}_p + \lambda\ \mathbf{f}_h$ is a solution for every $\lambda \in \mathbb{R}$. If we chose λ to be of a sufficiently large positive value then we can ensure that the negative components in $\mathbf{f}_p$ are dominated by the positive components of $\mathbf{f}_h$, and $\mathbf{f} \in \mathbb{R}^k \times \mathbb{R}_+^{n-k}$. The arguments above yield a simple algorithm to find *at least* one set of force targets that can generate a given external wrench. Also, as the external wrench is varied in the course of a manipulation task, this algorithm updates the force targets fast.

Yet another formulation of a closure grasp is via *form closure*. Recall that with each nominal point of contact we can also associate a twist system; they describe the degrees of freedom of the body local to that contact point. Thus a system with a set of contacts is free to move by a twist if and only if the virtual coefficient of any wrench and the twist is nonnegative, since otherwise the virtual work done by some wrench would be negative. This occurs when the twist is *reciprocal* to the bisense wrenches and *reciprocal* or *repelling* to the unisense wrenches. A set of twists (associated with the contacts) is said to constitute a *form closure* if and only if any arbitrary twist is resisted by the set of contacts. That is, the object is *totally constrained* with no degree of freedom left. Thus if $\mathbf{d}$ is an arbitrary twist then it must be *non-reciprocal* to some $\mathbf{w}_i$ $(i = 1, \ldots, k)$ or must be *contrary* to some $\mathbf{w}_j$ $(j = k+1, \ldots, n)$. Put another way, this is equivalent to saying that, for any arbitrary vector $\mathbf{d}' \in \mathbb{R}^6$, we have

$$\begin{aligned} \mathbf{w}_i \cdot \mathbf{d}' &= 0, \quad \text{for all } 1 \leq i \leq k \qquad \text{implies that} \\ \mathbf{w}_j \cdot \mathbf{d}' = \pi \mathbf{w}_j \cdot \mathbf{d}' &< 0, \quad \text{for some } k+1 \leq i \leq n, \end{aligned}$$

which, in turn is equivalent to the condition that

$$\mathbf{0} \in \text{int conv } (\pi \mathbf{w}_{k+1}, \ldots, \pi \mathbf{w}_n)$$

Thus, *force/torque closure* and *form closure* are equivalent; this idea seems to have been accepted by the researchers in the area. We also emphasize that this formulation has turned a problem in mechanics into a purely geometric problem, now amenable to many interesting techniques in convexity theory and computational and combinatorial geometry. This observation has been most fruitfully exploited in the work of [15].

Next we consider the case when the forces are monogenic. In this case we can associate a potential energy with each finger, and the potential energy of the object grasped by a set of fingers is simply the sum of the individual finger potential energies and the potential energy of the object. Thus the potential energy V of a grasp G is

$$V = \sum_i \int_{\sigma_{io}}^{\sigma_i} f_i(\sigma)\, d\sigma + V_o(\mathbf{d}_o, \theta_o),$$

where σ_i is the distance from a fixed origin to the prehension state, and σ_{io} is the distance to a non-prehension state. The first summand is the amount of work required to position each finger against finger forces from the free position of the finger to the prehension position on the object surface; $f_i(\sigma)$ is the force exerted by the i^{th} finger along the finger locus. The second summand is the potential energy of the object at the particular orientation, given by $\mathbf{d}_o$, the coordinate of the center of gravity of the object in fixed space, θ_o, the orientation of the object represented as angles between the coordinate axes and a fixed axis in the object; this may be assumed to be caused by some conservative field such as gravitational field.

For example, if we assume that each finger force is caused by the compliance of the finger (modeled as a linear spring) then the i^{th} finger force, f_i, is proportional to the i^{th} finger compression, σ_i:

$$f_i = k_i \sigma_i$$

where k_i is the i^{th} spring constant. So, the potential function is

$$V = \sum_{i=1}^{n} \frac{1}{2} k_i \sigma_i^2 + V_o.$$

When the object is in a grasp, the work function $U = -V$ is a scalar function of $[\mathbf{d}_o, \theta_o] = [d_{o1}, d_{o2}, d_{o3}, \theta_{o1}, \theta_{o2}, \theta_{o3}]$, and thus the infinitesimal virtual work $\overline{\delta w}$ is a true differential of U. Thus the virtual work is

$$\begin{aligned} \delta U &= \frac{\partial U}{\partial d_{o1}} \delta d_{o1} + \frac{\partial U}{\partial d_{o2}} \delta d_{o2} + \frac{\partial U}{\partial d_{o3}} \delta d_{o3} + \frac{\partial U}{\partial \theta_{o1}} \delta \theta_{o1} + \frac{\partial U}{\partial \theta_{o2}} \delta \theta_{o2} + \frac{\partial U}{\partial \theta_{o3}} \delta \theta_{o3} \\ &= [F_{o1}, F_{o2}, F_{o3}, \tau_{o1}, \tau_{o2}, \tau_{o3}] \cdot [\delta d_{o1}, \delta d_{o2}, \delta d_{o3}, \delta\theta_{o1}, \delta\theta_{o2}, \delta\theta_{o3}], \end{aligned}$$

where $F_o = [F_{o1}, F_{o2}, F_{o3}]$ and $\tau_o = [\tau_{o1}, \tau_{o2}, \tau_{o3}]$, are respectively the force and moment on the object. When the grasp is in equilibrium, the sum of all forces and moments acting on the object equals zero. This is equivalent to saying that the gradient of U (thus V) vanishes:

$$\nabla V = \mathbf{0}.$$

A grasp is called *stable* if when a relative position between the hand and the object deviates from a certain situation, a restoring force is generated by the fingers so that the relative position is brought back to the original situation. Thus a grasp is stable when the potential function reaches a local minimum, which occurs when the gradient of V, ∇V, is equal to zero and the Hessian matrix $\mathcal{H}$, of the second partial derivatives is positive definite.

In addition we can also define the *stiffness*, $\mathcal{K}$ (a 6×6 matrix) of a grasp to be the relationship between the force/torques applied to the grasp and the resulting motions:

$$[\delta F, \delta\tau]^T = \mathcal{K}[\delta\mathbf{d}, \delta\theta]^T.$$

The *compliance*, $\mathcal{C}$, is the inverse of the stiffness matrix. For a stable grasp, the restoring wrench

$$[\delta F, \delta\tau]^T = -\nabla V(\mathbf{d}, \theta) \approx \mathcal{H}|_{[\mathbf{d},\theta]}[\delta\mathbf{d}, \delta\theta]^T.$$

For small displacements, the compliance of the object is described by a stiffness matrix equal to the Hessian matrix. Thus the stability of a grasp is equivalent to the requirement that the stiffness matrix $\mathcal{K}$ is positive definite.

Next we consider various results in the area of grasping in greater detail.

4.1 Number of Fingers Required to Achieve a Grasp

In order to obtain a particular grasp on an object, it must be determined if that grasp is achievable. It is for this reason, researchers have studied the question of how many fingers (wrenches) are required to obtain certain grasps on the object.

Reuleaux and Somoff determined that the form closure of a two dimensional object requires at least four wrenches and of a three dimensional object requires at least seven, where the wrenches are normal to the surface of the object.

Mishra, Schwartz, and Sharir [15] gave the most general bounds on the number of fingers in the case of a *positive grip*; they also provided algorithms that find *at least one* such grip and run in time linear in the complexity of the object. Let d be the dimension of the object, and m_d be the degrees of freedom of a free object in d-dimensional space. (This is three in two dimension and six in three dimension). In summary their results show that the object can be held at equilibrium by a positive grip of $m_d + 1$ fingers (four fingers in two dimensions and seven fingers in three dimensions). If the object has no surfaces of revolution, then the object can be held with a closure grasp by a positive grip of $2m_d$ fingers (six fingers in two dimensions and twelve fingers in three dimensions), and any external wrench acting on the object can be balanced by a positive grip of m_d fingers. For the frictional case, their results can be generalized to show that the object can be held at equilibrium with $d + 1$ fingers (three in two dimension and four in three dimension), and can be held with a closure grasp also using $d + 1$ fingers.

Markenscoff, Ni, and Papadimitriou [13] proved that when there is no friction, for an object with a piecewise smooth boundary, four fingers are required to achieve form closure of a planar object, while twelve fingers are required for a three-dimensional object (but under certain conditions, only seven fingers are required). In all cases, however, form closure is not achievable if the object has a rotational symmetry. Under relaxed assumptions, when Coulomb friction is taken into account, three fingers are required in two dimensions and four fingers are required in three dimensions. Although the proof techniques used here are quite different from those used by [15], the results are quite similar.

4.2 Grip Transform

Salisbury [14] identifies the external and internal forces acting on the object in a grip matrix which is used to determine how to apply forces and control the motion and stiffness of the object. He shows that when a grasp *totally constrains* an object there are contact and frictional forces it can apply to the object while maintaining its equilibrium, and he then shows how to identify these forces. In the course of performing a task with a grasped object, the object will be subjected to external forces. In order to measure these forces, as well as the internal forces acting on the body, Salisbury defines the Grip Transform, $\mathcal{G}$, which like the Jacobian $\mathcal{J}$, is used to map forces between finger joints and grasped object coordinates. $\mathcal{G}$ is a square matrix whose dimension and rank is the number of contact wrenches acting on the object, and so it is constant for a particular grasp on an object.

As before, let $\mathcal{W} = [\mathbf{w}_1^T, \mathbf{w}_2^T, \ldots, \mathbf{w}_n^T]$ be a $6 \times n$ matrix of linear rank 6; each $\mathbf{w}_i$ is the screw coordinates of the principal wrench of the ith contact wrench system. Then the net wrench, $\mathbf{w}$, applied to the object is given by $\mathbf{w}^T = \mathcal{W}\mathbf{f}^T$, where $\mathbf{f}$ is a vector of contact wrench intensities. The grip transform $\mathcal{G}^{-T}$ can now be constructed by augmenting the $6 \times n$ matrix $\mathcal{W}$ with the $n-6$, n-element (orthonormal, linear) basis vectors of the homogeneous solution.

$$F^{*T} = \mathcal{G}^{-T}\mathbf{f}^T$$

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ \tau_x \\ \tau_y \\ \tau_z \\ \lambda_1 \\ \vdots \\ \lambda_{n-6} \end{bmatrix} = \begin{bmatrix} \\ \mathcal{W} \\ -\,-\,-\,-\,-\,-\,-\,- \\ \\ \mathbf{f}_h^{(1)} \\ . \\ . \\ . \\ \mathbf{f}_h^{(n-6)} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ . \\ . \\ . \\ . \\ . \\ f_n \end{bmatrix} .$$

The grip transform maps a set of forces and moments acting on the fingers, to external and internal forces acting on the fingers. Similarly, given a desired force we wish to exert on the object, $\mathcal{G}$ produces the forces and moments needed for each finger.

Additionally, $\mathcal{G}$ maps the finger velocities to the body's velocities:

$$V^{*T} = \mathcal{G}V^T$$

where $V^* = [v_x, v_y, v_z, \omega_x, \omega_y, \omega_z, \gamma_1, \ldots, \gamma_{n-6}]$, the body's linear velocity, angular velocity, and virtual velocities accounting for body deformations (non-zero values for γ_i's indicates that the grip has slipped or constraints have been violated) and V is a vector of twist intensities per unit time acting along the wrench axes at each contact. $\mathcal{G}$ provides a basis for applying forces and controlling small motions with an articulated hand.

We also mention a paper by Yoshikawa and Nagai ([25]), which introduces an intuitively appealing notion of internal forces, and addresses the problem determining *gripping forces* and *manipulating forces*. Their analysis is quite geometric, since the interpretation of gripping forces depend on the locus of feasible points of concurrence of the gripping forces. Unfortunately, the analysis is for three-finger frictional grips in the space, and does not seem to generalize to other cases.

4.3 Stability

Hanafusa and Asada ([5]) introduced the concept of stable prehension, similar to the one discussed earlier. They did their analysis in two dimensions using a robot hand with elastic fingers and rollers on the fingertips to eliminate friction. There are three coordinate systems used in the computation: x_o is the coordinate of the hand center, x_g is the coordinate of the sectional center of the object, and x_i are the finger positions for $i = 1,2,3$.

The two conditions for desirable hand position are

1. *Stability in the horizontal plane*: This amounts to deriving a local minimum for the potential function $V(x, y, \theta)$.

2. *Force and moment balance in the vertical direction*: The friction due to the finger forces must be large enough to support the weight of the object. Based on the moment balance condition, Hanafusa and Asada adopt the following heuristic:

$$|x_o - x_g| \leq R,$$

   where R is an appropriate positive number.

The computation algorithm has two steps:

1. Calculate x_g and examine the local minimum of U with respect to θ by keeping the hand center on the section center. If $U(\theta; x_o = x_g)$ has more than one value, then record all the minimum points, θ_k. At $\theta = \theta_k$, the moments in the horizontal plane are balanced.

2. Approximate the local object properties by circular arcs in the vicinities of $\theta = \theta_k$ and minimize U with respect to x and y by keeping the moment balance.

Nguyen [17, 18] extended this work in finding stable grasps in the plane and in three dimensions. His criteria for stability are that the object be in static equilibrium and the grasp be force closure. For the 2D case, he considered objects that were planar polygons, multiple point contacts without friction, and the fingers were modeled as virtual springs with controllable stiffness.

In three dimensions [18], the contacts are modeled as frictionless point contacts, hard-finger contacts, or soft-finger contacts with friction. The fingers are virtual springs, and each virtual spring is a set of independent linear and angular springs. These correspond to the wrenches defined by Salisbury.

As a grasped object is displaced by an infinitesimal twist $\widehat{\mathbf{t}} = [\delta, \mathbf{d}]$, the new position of the contact of the i^{th} finger is $\mathbf{p}'_i$, given by $\mathbf{p}'^T_i = \text{Rot}(\delta_x, \delta_y, \delta_z)\mathbf{p}^T_i + \mathbf{d}^T$ where

$$\text{Rot}(\delta_x, \delta_y, \delta_z) \approx I + [\delta\times] + \frac{1}{2}((\delta\delta^T) - (\delta \cdot \delta)I)$$

$$\approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\delta_z & \delta_y \\ \delta_z & 0 & -\delta_x \\ -\delta_y & \delta_x & 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} -(\delta_y^2 + \delta_z^2) & \delta_x \delta_y & \delta_z \delta_x \\ \delta_x \delta_y & -(\delta_z^2 + \delta_x^2) & \delta_y \delta_z \\ \delta_z \delta_x & \delta_y \delta_z & -(\delta_x^2 + \delta_y^2) \end{bmatrix}$$

The compression, σ_i, of a linear spring, k_i due to a twist of the object is equal to the initial compression plus the dot product of the line of action of the spring and the twist displacement of the object, i.e. $\sigma_{io} + \mathbf{k_i} \cdot (\mathbf{p'_i} - \mathbf{p_i})$. Using the formula above, the effective compression can be approximated by

$$\sigma_{io} + [\mathbf{k}_i, \mathbf{p}_i \times \mathbf{k}_i] \cdot [\mathbf{d}, \delta] + \frac{1}{2} [k_{ix}, k_{iy}, k_{iz}]((\delta \delta^T) - (\delta \cdot \delta) I) \begin{bmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{bmatrix}$$

For an angular spring, the twist will have the same angular part, but a different linear part, so

$$\widehat{\mathbf{t}} = [\delta, (\mathbf{d} - (\mathbf{p}_i \times \delta))]$$

and the angular displacement is then

$$\sigma_i = \mathbf{k_i} \cdot \delta = [0, \mathbf{k_i}] \cdot [(\mathbf{d} - (\mathbf{p}_i \times \delta)), \delta]$$

The potential function, V, is now

$$\begin{aligned} V &= \sum_{i=1}^{n} \frac{1}{2} k_i \sigma_i^2 \\ &= \frac{1}{2} [\sigma_1, \ldots, \sigma_n] \begin{bmatrix} k_1 & & \\ & \ddots & \\ & & k_n \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_n \end{bmatrix} \end{aligned}$$

The grasp is in equilibrium when the gradient of its potential function is zero. In matrix form: $\nabla V|_{\mathbf{x}=0} = \sum_{i=1}^{n} k_i \sigma_{io} \widehat{\mathbf{k}}_i = 0$

4.4 Stiffness and Compliance Control

Salisbury [14] showed how to control the stiffness in order to obtain a stable grasp. The stiffness of a link determines what forces will be acting on the link if it is displaced by a small amount. $F = \mathcal{K}_C \mathbf{d}$ are the forces required to deflect a generalized spring by a displacement of $\mathbf{d}$, where $\mathcal{K}_C$ is the stiffness matrix in Cartesian coordinates. The energy stored in the system is $V = \frac{1}{2} \mathbf{d} \mathcal{K}_C \mathbf{d}^T$, which will be positive if the stiffness matrix is positive definite. The stiffness matrix in terms of joint coordinates, $\mathcal{K}_\theta = \mathcal{J}^T \mathcal{K}_C \mathcal{J}$, is positive definite if $\mathcal{K}_C$ is, since positive definiteness is preserved under congruence transformations, thus any perturbation of the joints will result in joint forces that will tend to return them to their original positions.

To obtain the finger joint coordinates required to achieve a desired stiffness of the grasped object, $\mathcal{K}_C$ must be augmented to include the independent internal forces present in the grasp which will be used to stabilize the grasp and preserve positive contact forces at the fingers:

$$\mathcal{K}_C = \text{diag } (k_x, k_y, k_z, g_x, g_y, g_z, k_1, \cdots, k_p)$$

The first six diagonal elements are the translational and rotational stiffnesses of the grasped object, while the last p elements account for the internal stiffness. A composite Jacobian matrix, $\mathcal{J}^*$ is then formed. $\mathcal{J}^*$ is a block-diagonal matrix where the matrices along the diagonal are the individual finger Jacobians. Thus, in order to apply a generalized force $[F, \tau]$ to the object, the required finger joint torques are given by, $\tau'^T = \mathcal{J}^{*T}\mathcal{G}^T\mathbf{f}^T$, and $\mathcal{K}_\theta = \mathcal{J}^{*T}\mathcal{G}^T\mathcal{K}_C\mathcal{G}\mathcal{J}^*$.

4.4.1 Diagonalizing the Stiffness Matrix

Recall that, for small displacements, the compliance of the object is described by a stiffness matrix equal to the Hessian matrix. Nguyen solves for the stiffness matrix for the cases when the Hessian matrix is diagonalizable. There are two special cases where the Hessian matrix is diagonalizable. This occurs when the center of compliance of the object is at the common intersection of the lines of action of the springs and when the center of compliance is such that the weighted sum of the virtual springs is zero. He shows for this second case how to ensure that the stiffness matrix will be positive definite by choosing the center of compliance of the object.

Extending this to three dimensions, the stiffness matrix $\mathcal{K}$ is the Hessian matrix $\mathcal{H}$ of V at the equilibrium configuration.

$$\mathcal{K} = \mathcal{K}_S + \mathcal{K}_P$$

where $\mathcal{K}_S$ depends on the spatial vectors of the l linear springs and the $n-l$ angular springs and $\mathcal{K}_P$ depends on the angular stiffness matrix:

$$\begin{aligned}
\mathcal{K}_S &= [\widehat{\mathbf{k}}_1^T, \ldots, \widehat{\mathbf{k}}_n^T] \begin{bmatrix} k_1 & & \\ & \ddots & \\ & & k_n \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{k}}_1 \\ \vdots \\ \widehat{\mathbf{k}}_n \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{k}_1^T & \ldots & \mathbf{k}_l^T \\ (\mathbf{p}_1 \times \mathbf{k}_1)^T & \ldots & (\mathbf{p}_l \times \mathbf{k}_l)^T \end{bmatrix} \begin{bmatrix} k_1 & & \\ & \ddots & \\ & & k_l \end{bmatrix} \begin{bmatrix} \mathbf{k}_1 & (\mathbf{p}_1 \times \mathbf{k}_1) \\ \vdots & \vdots \\ \mathbf{k}_l & (\mathbf{p}_l \times \mathbf{k}_l) \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{0}^T & \ldots & \mathbf{0}^T \\ \mathbf{k}_{l+1}^T & \ldots & \mathbf{k}_n^T \end{bmatrix} \begin{bmatrix} k_{l+1} & & \\ & \ddots & \\ & & k_n \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{k}_{l+1} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{k}_n \end{bmatrix}, \\
\mathcal{K}_P &= \pm \begin{bmatrix} [0] & [0] \\ [0] & \mathcal{K}_{P\delta} \end{bmatrix},
\end{aligned}$$

where $\widehat{\mathbf{k}}_i$ is a spatial vector representing the configuration of a linear spring (if $1 \le i \le l$) or an angular spring (if $l+1 \le i \le n$).

The sign depends on whether or not friction is present (+ denotes no friction, and that the fingertips slide on the surface of the object, $-$ denotes presence of friction, and that the fingers stick to the point of contact). $\mathcal{K}_{P\delta}$ corresponds to the position of the angular stiffness matrix and is equal to $(\sum_{i=1}^{l} f_{io}(\mathbf{p}_i \cdot \mathbf{k}_i))I$; where f_{io} corresponds to the contact force due to the ith linear spring. If $\mathcal{K}$ is positive definite, then the grasp is stable.

4.4.2 Stiffness, Stability, and Resistance to Slipping

In his analysis, Cutkosky [2] is mainly concerned with robots in a manufacturing domain. He assumes that the stiffness of each finger is known. The motions and force changes which result from displacement of the object are computed, and are then used to determine the overall stiffness of the grasp, the ability of the grasp to resist slipping, and the ability of the grasp to return to equilibrium in the event of disturbances.

The fingers are elastic and the object is rigid. Motions are considered to be small, so that the transformations can be linearized, and only motions with respect to the hand are considered. The analysis is static, and while it does not try to find the optimum grip for a given task, it does provide a mechanism for evaluating certain properties of the grip. There are three main criteria used in choosing an appropriate grip:

1. *Stiffness*: Grasp stiffness depends on the stiffness of the fingers and their arrangement about the object. A stiff grip would be useful in manipulating objects at high speeds. The best grip is the one which best matches the achievable finger stiffnesses to the task requirements.

2. *Stability*: Since this analysis is concerned with small motions, the stability criterion considers whether a grip is *infinitesimally stable*, i.e. whether the grip will return to its original position if the object is displaced by an arbitrarily small amount.

3. *Resistance to slipping*: The grip should be able to resist the greatest possible applied force or torque before any of the fingers slip. This would be desirable for manipulating objects at high speeds.

Cutkosky outlines a five step procedure for establishing these grip properties:

1. Displace the object by an arbitrary small amount.

2. Determine the resulting motions of the fingers. These will depend on finger geometries, contact types, and stiffnesses.

3. Determine the change in forces at the contact areas that resulted from the motion. These changes are caused by changes in the grip and the restoring forces due to the finger stiffnesses.

4. Compare the new forces at the contact areas with the maximum forces that the contacts can endure without slipping, and determine whether the normal forces would become negative at any of the fingers (this corresponds to a loss of contact).

5. Compare the new forces and torques on the object with the original ones and with the displacement of the object to determine the stiffness and infinitesimal stability of the grip.

In order to carry out the computational steps, one needs to express the interaction between grasping forces and small motions of the object. Thus it necessary to determine how the forces applied by the fingers affect the grasp force and how small motions of the object influence the resulting motions of the fingers. Cutkosky shows that the effective stiffness of the object being grasped is a function of the servo parameters, fingertip models, and small changes in the grasp geometry as the object is perturbed by external forces.

As others, Cutkosky models the grasped object, the set of fingers with the points of contact as independent rigid objects. The configuration spaces for the fingers (in terms of the joint variables), the contacts and the objects are expressed in separate coordinate systems. Note that the configuration space of the object is a six-dimensional Euclidean space, whereas the configuration spaces of the contacts and the fingers are determined, respectively, by the wrench systems associated with the contacts and by the independent finger-joints. The relation between the contact forces and the external force on the object is determined by the grasp transforms, and the relation between the joint forces and the contact force is determined by each finger Jacobian. By the principle of virtual work, similar relations exist between the instantaneous kinematics (i.e. velocities or infinitesimal displacements) of the joints, contact points and the object. Similarly, assuming that the fingers do not slip relative to the object when the object moves infinitesimally, an equivalent stiffness expression can be given, relating the external force on the object to the displacements of the object. During the computation of steps 2 and 3, however, we need to compute both forward and inverse transforms, and since all these systems are not *exactly determined*, some care must be given to the inverse transforms. In particular, we need to consider the following three cases:

1. The motion of the object exactly determines the motion of the finger. For this case, the Jacobian can be inverted to obtain the inverse displacement and force relations.

2. The motion of the finger is underdetermined. The remaining undetermined elements may be computed by adding constraints such as those requiring that the potential energy be minimized.

3. The motion of the finger is overdetermined. The motion of the finger is limited. However, it is also possible that for some motions there may be undetermined elements that can be solved for as in case 2.

5 Discussion of the Mechanical and Physical Approach

Much of the research has provided useful tools that may bring us closer to *useful* hands. Salisbury and Cutkosky studied contact models, and how they affect the freedom of the object. Mishra, Sharir, and Schwartz as well as Markenscoff, Ni, and Papadimitriou calculated how many fingers are needed to obtain grasps on certain objects. Salisbury introduced the notion of the Grip Transform. He used it to neatly describe the forces present in a grip and what forces can be applied to the grip without disturbing the equilibrium. Hanafusa and Asada introduced the notion of stability, but only determined stable grasps for a very simplified model. The analysis works for the case of no friction, but has limited usefulness since friction is an important consideration. Nguyen addressed the stiffness of objects grasped by virtual springs. He added the stiffness consideration to Hanafusa and Asada's work, and also extended it to three dimensions. However, in trying to generalize the calculations, they became too complex, so he only dealt with limited simplified cases. Cutkosky obtained other measures for grasps in addition to stability. He showed how to measure the stability, stiffness, and resistance to slippage, which are other important considerations for choosing a grasp.

For humans, grasping an object seems like a simple task, yet analyzing the details involved shows us that grasping is very complex and there are many issues to be considered. The primary focus until now has been establishing statically stable grips, such as was done by Hanafusa and Asada and also Nguyen, although researchers are now dealing with finding grasps suitable to the performance of tasks.

What emerges from all this is the importance of finding a stable grasp. A lot of work has been put into this problem, but most of the researchers seem to come up with the same mathematical calculations (much of which were omitted in the descriptions). The main idea seemed to be to obtain a stable grasp by identifying force relationships, programming the stiffness of the fingers, and reducing the degrees of freedom of the object. Jacobians were obtained and calculations were done using standard linear algebra. However, all these calculations included constraints which were added to obtain results, yet it remains unclear how to determine some of these constraints and why they were chosen.

The most unsatisfactory situation in the whole area concerns the problem of synthesizing a grasp. While the physiological approach has succeeded in a limited way to address these issues, there is little or no progress by the researchers using analytical techniques. The work of [15] does provide an algorithm to synthesize a positive grip, but this is only worked out in the case of an idealized hand, and no effort has been made to come up with a grip that satisfy additional criteria of "goodness." Techniques to synthesize grasps and to integrate these grasps into homogeneous manipulations must be developed, and this is where ideas from both approaches can be used together.

6 Conclusions

Although many of the problems involved in grasping are far from being solved, some progress has been made: problems have been identified and tools have been established.

In studying the physiological aspects of grasping, researchers developed numerous grasping taxonomies. These taxonomies have been used to develop robot grippers and to assist robot hands in grasping certain objects. Since human hands are efficient and multi-purpose grippers, studying how humans grasp objects can hopefully explain the science of grasping. While some systems have been built which use taxonomies to choose an appropriate grip, this does not always work due to unusually shaped objects and additional unknown factors. Therefore, the physical and mechanical properties must be studied.

The literature is more extensive in the areas of identifying contact types and their effects and restrictions on grips. Salisbury's grip transform is an important tool for describing force relationships between the fingers and the grasped object. The main results are the identification of certain properties of grips, such as the potential energy, stability, and compliance, and the establishment of the relationships between forces and movements of the hand and the object.

The physiological and mechanical approaches are complementary. The physical and mechanical results are needed to understand the low level elements involved in grasping. This includes contact types, effects of friction, compliance, slipping, and what is involved in obtaining a stable grasp. However, for higher level elements such as task planning, studying how human hands grasp objects and perform tasks is more useful.

For the purpose of performing a task, it might not be sufficient to have a stable grasp. Often, there are many different possible stable grasps, and so we will have to choose which one, if any, is appropriate for the task to be performed. There has been little success in evaluating the suitability of a grasp to perform a particular task in terms of its mechanical properties, and therefore, these grasp taxonomies (and expert systems) can better assist the dextrous hand in choosing a grasp.

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