

Assignment 3 Graph Theory

Due: October 30th, 2019 at the end of class.x

For any doubts or queries regarding assignment please attend office hours on Monday 9:30-10:30 in CIWW 412. The assignment can be written or typed. Please write your name and netID on the assignment.

1. The government requires a group of cities that are randomly picked for security reasons. It requires that in the group there must be at least one subset of 3 cities that either are all mutually connected by road or they all are mutually disconnected. What is the minimum number of cities that the government needs to pick? (Assume that given any two cities, they must be either connected or disconnected)
2. Describe page rank algorithm very briefly **in your own words**. (50-100 words)
3. Find the number of spanning trees for the complete graph on 4 vertices.
(Bonus 1 mark if done by **Kirchhoff's theorem**)
4. You create a picture and want it to be seen by everyone in the world. You share it with 3 people on the first day. They each in turn share it with 3 new people the next day. So every person who receives it shares it with 3 new people the day after they receive the photo.
(Let a new person be someone who is receiving the message for the first time.)

For some reason there is an internet blackout for 2 days worldwide in one of the subsequent days (The blackout can take place any day after you shared the picture for the first time and before everyone in the world receives it). During the blackout $\frac{2}{3}$ rd of the people who were supposed to forward the photo the next day stop forwarding the photo. They are not considered new people, they just don't forward the message anymore. The remaining people who received the photo the day before the blackout, forward the photo the day after the blackout ends. And this continues

Finally the message reaches everyone in the world. How many days longer did it take to reach the world compared to the case where there was no internet blackout and thus no loss of those $\frac{2}{3}$ people who stopped forwarding.

Hint: If there was no blackout the total number of days will be x such that $\sum_{i=0}^{x-1} 3^i < P \leq \sum_{i=0}^x 3^i$

where P is the population of the world.