

LECTURE #3STRONG TRIADIC CLOSURE.

- In a social network, let  $f_1$  and  $f_2$  be two close friends of yours...
  - ⇒ They are connected by strong ties to you.
  - ⇒ Then they ( $f_1$  and  $f_2$ ) belong to a  $k$ -clan (for small  $k$ ) with high prob.
  - ⇒ Then, it is likely that  $f_1$  and  $f_2$  are acquaintances and are connected to each other by weak ties.

[ At least your social network may recommend that  $f_1$  and  $f_2$  explore connecting with each other. ]

$$\Pr[(v, w) \in E \mid (u, v) \in E_s \wedge (u, w) \in E_s] > \Pr[(v, w) \in E].$$

## TRIADIC CLOSURE PROPERTIES

- ◊ If  $f_1$  and  $f_2$  have a large subgroup of common friends (which include you)
  - ⇒ Then it is ~~possible~~ probable that they are acquaintances.
  - ⇒ The probability monotonically increases with the size of the set of mutual friends.
- ◊ In a connected social network, we expect to see lots of  $K_3$ 's (Cliques of size 3).

### Empirical Results.

"Strength of Weak Ties" (1973)

Mark Granovetter: (American Sociologist, Currently at Stanford Univ.)

- ◊ "Weak ties enable reaching populations and audiences with much higher efficiency than what is achievable or accessible via strong ties."

WHY?

◇ Granovetter's PhD dissertation:  
(Dept of Social Relation, Harvard Univ.)

"Getting a Job"

Experiment:

Location: Newton, MA.

Subjects: 282 professional, technical & managerial workers.

Result: N = 54 = # individuals (out of 282) who found jobs through personal contacts.

Strong Ties (16.6%)

Weak Ties (83.4%)

Occasional Contacts (55.6%)

Rare Contacts (27.8%)

# AUGMENTED GRAPH

Def<sup>n</sup>: Consider an "augmented" graph (undir.)

$$G = (V, E, E_s)$$

in which

$$E_s \subseteq E \subseteq V \times V.$$

$E$ : The edges/ties. }  $E \setminus E_s$ : The weak ties.  
 $E_s$ : The strong ties.

$\Rightarrow (u, v) \in E \Rightarrow u$  and  $v$  are friends  
(acquaintances + close friends)

$(u, v) \in E_s \Rightarrow u$  and  $v$  are close friends.

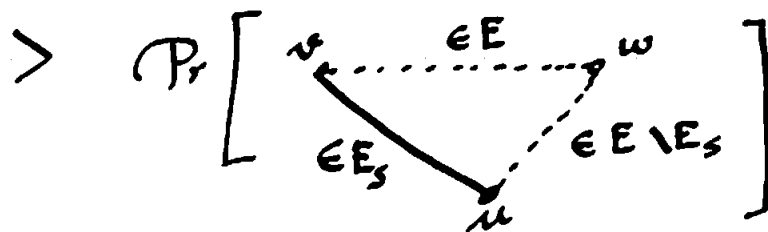
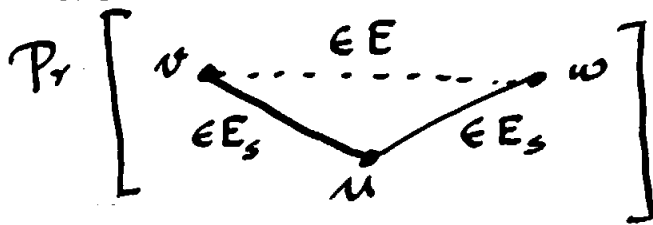
The strong triadic closure property

if  $(u, v) \in E_s$  and  $(u, w) \in E_s$   
then  $(v, w) \in E$  a.s.

$$\Leftrightarrow \Pr[(v, w) \in E \mid (u, v) \in E_s \wedge (u, w) \in E_s] > \Pr[(v, w) \in E]$$

$\Rightarrow$  Probability Raising in Social Network.  
(Influence vs Social Influence)

$$(I) \Pr [(v,w) \in E \wedge (u,v) \in E_s \mid (u,w) \in E_s] \\ > \Pr [(v,w) \in E \wedge (u,v) \in E_s \mid (u,w) \in E \setminus E_s]$$



(II) Consider a new relation  $R$

$\{(u,v) \in R\}$  = Event  $u$  obtained a job through a "recommender"  $v$ .

[  $v$  recommended  $u$  to  $w$ ;  
 $w$  verified  $u$  independently;  
 $w$  determined whether  $u$  is a suitable candidate. ]

$$\begin{aligned}
 & \Pr [(u, v) \in R \mid (u, v) \in E_s] \\
 & \approx \Pr [(u, w) \notin E \wedge (v, w) \in E_s \mid (u, v) \in E_s] \\
 & < \Pr [(u, w) \notin E \wedge (v, w) \in E_s \mid (u, v) \in E \setminus E_s] \\
 & = \Pr [(u, v) \in R \mid (u, v) \in E \setminus E_s]
 \end{aligned}$$

①

$u$  = Applicant;  $v$  = Recommender  
 $w$  = Verifier (Employee)

Strong Ties:

$$(u, v) \in E_s \wedge (v, w) \in E_s$$

$$\Rightarrow (u, w) \in E$$

$w$  knows about  $u$  and

can use information in addition

to what's provided in  $v$ 's recommendation.

②

Weak Ties:

$$(u, v) \in E \setminus E_s \wedge (v, w) \in E_s$$

$$\Rightarrow (u, w) \notin E$$

$w$  will go by  $v$ 's recommendation only.