Social Networks

Lecture #8

Probability Distributions.

- **Bernoulli** $X \sim \text{Bernoulli}(p)$

  $X \in \{0, 1\}$

  $\Pr[X=1] = p \qquad \Pr[X=0] = (1-p) = q$

  $\boxed{\begin{array}{l}\mu = p \\ \sigma^2 = p - p^2 \\ \quad = pq\end{array}}$

  $\Pr[X=k] = \binom{1}{k} p^k q^{1-k} = p^k q^{1-k}$

- **Binomial** $X \sim B(n, p) \quad X \in \{0, 1, 2, \cdots, n\}$

$$\Pr[X=k] = \binom{n}{k} p^k q^{n-k}$$

$\boxed{\begin{array}{l}\mu = np \\ \sigma^2 = npq\end{array}}$

In n independent trials,
exactly k successes are observed.

- **Poisson** $X \sim \text{Poisson}(\lambda)$

$$\Pr(X=k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$$

Average rate of an event per unit time $= \lambda$
Exactly k events are observed in a unit time

$$X \sim B\left(n, \frac{\lambda}{n}\right)$$

$$\Pr(X=k) = \binom{n}{k} \left(\frac{\lambda}{n}\right)^k \left(1 - \frac{\lambda}{n}\right)^{n-k}$$

$$Pr(X=k) = \frac{n^{(k)}}{k!} \frac{\lambda^k}{n^k} \left(1-\frac{\lambda}{n}\right)^n \left(1-\frac{\lambda}{n}\right)^{-k}$$

$$= \frac{e^{-\lambda}}{k!} \lambda^k \frac{n^{(k)}}{n^k} \frac{1}{(1-\frac{\lambda}{n})^k}$$

$$\lim_{n\to\infty} \longrightarrow \frac{e^{-\lambda}\lambda^k}{k!} \qquad \mu = \lambda \quad \sigma^2 = \lambda$$

$$\mu \approx n.\frac{\lambda}{n} \quad \sigma^2 \approx n.\frac{\lambda}{n}\left(1-\frac{\lambda}{n}\right)$$

- Gaussian $\quad X \sim \mathcal{N}(\mu, \sigma^2)$

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mu = \mu$$
$$\sigma^2 = \sigma^2$$

- CLT (Central Limit Theorem)

$x_1, \ldots, x_n$ i.i.d. with $E(x_i) = \mu$, $Var(x_i) = \sigma^2$

Then

$$\frac{\sum_{i=1}^{n} x_i - n\mu}{\sqrt{n}\,\sigma} \sim \mathcal{N}(0,1)$$

RANDOM GRAPH

Erdös - Renyi

$G(n,p)$ Model

A graph $G = (V, E)$ is constructed by connecting every pair of nodes uniformly randomly.

$$\forall_{u,v \in V} \quad [(u,v) \in E] \sim \text{Bernoulli}(p) \quad \text{i.i.d.}$$

$$|V| = n \qquad \langle |E| \rangle = \binom{n}{2} p$$

$$d(v) \sim \text{Binomical}(n-1, p)$$

$$\left.\begin{array}{l} \bar{d} = (n-1)p = \lambda = \text{const.} \\ d(v) \sim \text{Poisson}(\lambda) \end{array}\right\} \text{Poisson Approximation}$$

Phase Transition (0-1 Laws)

$$p < \frac{(1-\varepsilon)\ln n}{n} \qquad G(n,p) = \text{Disconnected a}$$

$$p > \frac{(1+\varepsilon)\ln n}{n} \qquad G(n,p) : \text{Connected a.s.}$$