SOCIAL NETWORKS

Sept 22 2015

LECTURE # 6

COHESIVE SUBNETWORKS

A graph is complete (K_n) if all its vertices are adjacent to all others.

$$\forall_v \; d(v) = n-1$$

$$\sum d(v) = n(n-1) = 2|E|$$

$$|E| = \frac{n(n-1)}{2} = \binom{n}{2}$$

$$\bar{d} = (n-1)$$

$$\text{Density} = \frac{\bar{d}}{(n-1)} = 1.$$

- If a social network has many "well-connected individuals" then $\bar{d} = O(n)$
  - ⇒ The network is "dense."
  - ⇒ $\text{Density} = \frac{O(n)}{(n-1)} = O(1)$

- Finding and connecting to "well-connected" subnetworks is beneficial.

⇒ CLIQUE

CLAN

CLUB

(20)

A CLIQUE is a maximal complete subgraph.

Cohesive Subnetworks.

K-Clique: A k-clique S of a graph is a maximal set in which all pairs of vertices are at most k-distant away

$$\forall_{u,v \in S} \quad d(u,v) \leq k.$$

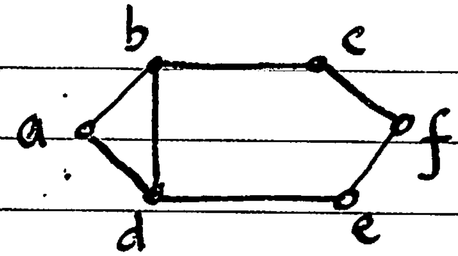
$$\boxed{1\text{-clique} = \text{clique}}$$

K-club: A k-club S of a graph is a maximal set such that subgraph $\langle S \rangle_G$ induced by S has a diameter at most k.

$$\delta(\langle S \rangle_G) = k.$$

$$\forall_{u,v \in S} \quad d_{\langle S \rangle_G}(u,v) \leq k.$$

k-clan = k-clique $\cap$ k-clan.



$\{a,b,d\}$ = 1-clique

$\{a,b,c,d,e\}$ = 2-Clique
$\neq$ 2-clan
($\neq$ 2-club)

$d(c,e) = 2$ (thru f)
$d_s(c,e) = 3$

$\{b,c,d,e,f\}$ = 2-clan (= 2-clique = 2-club)

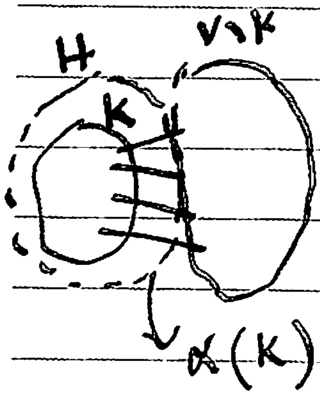
$\{a,b,c,d\}$ = 2-club ($\neq$ 2-clique)
$\neq$ 2-clan

k-Clique $\rightarrow$ k-clan $\leftarrow$ k-club

LS-Sets.

Let H be a set of nodes in $G=(V,E)$ and let K be a proper subset of H

$$K \subsetneq H$$



$$V \setminus K = \{v \in V \mid v \notin K\}$$

$$\alpha(K) = \left|\{\{u,v\} \in E \mid u \in K \wedge v \in V \setminus K\}\right|$$

= dentoes the number of edges linking members of K to $V \setminus K$.

H = An LS-set of G if

$$\forall_{K \subsetneq H} \quad \alpha(K) > \alpha(H)$$

Members of an LS-set have more ties with the "insiders" (other members of H) than the "outsiders" (members of $V \setminus H$).

Random Graphs

Probability Theory (+ Graph Theory)

(Discrete) Probability space $= (S, \mathcal{F}, \mathbb{P})$

S = Sample Space (= non-empty & countable)
$\mathcal{F}$ = Sigma Algebra
[Collection of all subsets of S
$\mathcal{F} = 2^S$]
The elements of $\mathcal{F}$ are events

$\mathbb{P} : \mathcal{F} \to \mathbb{R}$ = Probability measure with following properties.

(1) $\forall_{A \in \mathcal{F}} \quad \mathbb{P}(A) \in [0,1] \qquad \mathbb{P}(S) = 1$
$\mathbb{P}(\phi) = 0$

(2) $A_{i \in I}$ is a countable set of events that are pairwise disjoint
$\forall_{i \neq j \in I} \quad A_i \cap A_j = \phi$

$$\mathbb{P}\left(\bigcup_{i \in I} A_i\right) = \sum_{i \in I} \mathbb{P}(A_i)$$

Uniform Probability Space.

$$\mathbb{P}(A) = \frac{|A|}{n}$$

Lemma. $(S, \mathcal{F}, \mathbb{P})$ = A probility space.

$A, B \in \mathcal{F}$.

(1) $\mathbb{P}(\phi) = 0$

(2) $\mathbb{P}(S \setminus A) = 1 - \mathbb{P}(A)$

(3) $A \subseteq B \Rightarrow \mathbb{P}(A) \leq \mathbb{P}(B)$

(4) If $(A_i \in i \in I)$ = a countable set of events then

$$\mathbb{P}\left(\bigcup_{i \in I} A_i\right) \leq \sum_{i \in I} \mathbb{P}(A_i)$$

If $\mathbb{P}(B) > 0$ then the conditional probability that A occurs given B is

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}.$$

Event A and B are independent if

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\, \mathbb{P}(B)$$

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A)\mathbb{P}(B)}{\mathbb{P}(B)} = \mathbb{P}(A)$$

$$\wedge \quad \mathbb{P}(B \mid A) = \mathbb{P}(B)$$

A set of events $(A_i \mid 1 \leq i \leq n)$ is mutually independent if

$$\mathbb{P}\left(\bigcap_{i=1}^{n} A_i\right) = \prod_{i=1}^{n} \mathbb{P}(A_i) \quad \text{or} \quad \mathbb{P}(A_i \mid A_1 \cdots A_{i-1} A_{i+1} \cdots A_n) = \mathbb{P}(A_i)$$