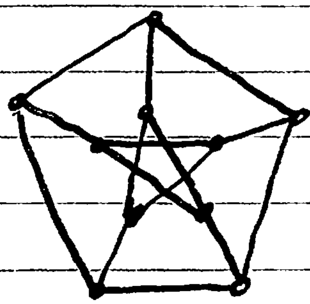
Social Networks

Sept 15 2015

Lecture #4



Peterson Graph.

$\forall_{v \in V} \; d(v) = 3 = \bar{d}$

Regular Graph.

$|V| = 10 \quad |E| = 15$

$\sum_{v \in V} d(v) = 3 \times 10 = 30 = 2 \times 15$

Given a vertex $u \in V$,
define its <u>neighbor set</u>

$$N(u) = \{x \mid (x,u) \in E\}$$

= Set of vertices joined to u

$$N^c(u) = \{\underset{\neq u}{x} \mid (x,u) \notin E\}$$

= set of vertices distinct from u and not joined to u.

$$V = \{u\} \cup N(u) \cup N^c(u)$$

<u>Def^n</u>: $S \subseteq V$, define $\langle S \rangle_G$ = Subgraph of G induced by S

$$\langle S \rangle_G = \langle S, E(G) \cap S \times S \rangle$$

= Graph with vertices S and with two vertices joined in $\langle S \rangle_G$ iff they are joined in G.

* A subgraph of G is a graph H s.t.

$V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

A spanning subgraph of G is a subgraph H s.t.

$V(H) = V(G)$. Note $E(H) \subseteq E(G)$.

$\delta(G)$: The minimum degree of G.

$= \min_{v \in V} d(v)$.

$\Delta(G)$ = The maximum degree of G

$= \max_{v \in V} d(v)$.

$\sum_{v \in V} d(v) = 2|E| \Rightarrow$ The number of odd degree vertices in a graph is even.

$\forall_{v \in V}\ d(v) = k \Rightarrow G =$ k-regular graph.

$k = \text{odd} \Rightarrow |V| = \text{even}$.

$\Rightarrow$ Graph is of even order.

WALK : A walk in a graph consists of an alternating sequence of vertices and edges:

$$x_0 e_1 x_1 \dots e_t x_t$$

$$\forall_{1 \le i \le t} \quad e_i = (x_{i-1}, x_i)$$

$$\boxed{x_0 x_t\text{-walk}}$$

A walk is closed, if $x_0 = x_t$; otherwise, open.

A <u>path</u> is an <u>open</u> <u>walk</u> with no repeated vertex.

A <u>cycle</u> is a <u>closed</u> <u>walk</u> with no repeated vertex

The number of edges in a walk (path, cycle) is its length.

n: W_n, P_n, C_n.

<u>Defn</u>: The <u>girth</u> of a graph G, $g(G)$ is the minimum length of a cycle in a graph.

<u>Lemma</u>: (i) If G is a graph, then G contains a path of length $\delta(G)$.

(ii) If $\delta(G) \geq 2$, then
$$g(G) \geq \delta(G)+1.$$

proof.

(i) $u_0, u_1, \dots, u_r = P =$ A path of max length r in G.

$\Rightarrow N(u_r) \subseteq \{u_0, u_1, \dots, u_r\}$ (why?)

$\Rightarrow \delta(G) \leq r$

(ii) Vertex joined to u_0 with min. index.

$u_0 - u_m - \cdots u_m - \cdots - u_r$

$u_m \cdots u_n \cdots u_r - u_m$

= cycle of length $\geq \delta(G)+1$

Petersen Graph

$$\delta(G) = \Delta(G) = 3$$

$$g(G) = 5 \qquad g(G) \geq \delta(G)+1.$$

◇ A graph is connected if for each pair of vertices, there is a path connecting them

$$u \rightsquigarrow u; \quad u \rightsquigarrow v \Rightarrow v \rightsquigarrow u \;\&$$

$$u \rightsquigarrow v \wedge v \rightsquigarrow w \Rightarrow u \rightsquigarrow w.$$

✱ The relation "connectivity" is an equivalence relation.

⇒ The <u>equivalence</u> classes are the connected components of G.

◇ The (geodesic)-distance between two vertices is the length of the shortest path connecting them.

$d(u,v)$ = Geodesic distance between u and v.

◇ The <u>maximal</u> geodesic distance in a graph is its <u>diameter</u> $\mathcal{D}(G)$

$$\forall_{u,v} \quad d(u,v) \leq \mathcal{D}(G).$$