SOCIAL NETWORKS

Sept 10 2015

LECTURE # 3

- An edge
$$e = (u,v) \in E$$
(where $E \subseteq V \times V$)
is described by the unordered pair of vertices (players), which serve as edge's end-points.

- Two vertices u and v are <u>adjacent</u> if
$$\exists_{e \in E}\ e = (u,v)$$
connecting u and v.

- NOTATION: $|V| = n$ = # vertices
$|E| = m$ = # edges.

$$m \leq \binom{n}{2} = \frac{n(n-1)}{2}$$

n = # ways to choose u

$n-1$ = # ways to choose $v\ (\neq u)$

identifying edge
$(u,v) \equiv (v,u)$ [SYM]

8

STRICT GRAPH:

No self-loop

$\forall u \quad (u,u) \notin E$

No multi-edge

$\forall e_1, e_2 \quad e_1 = (u_1, v_1) \equiv e_2 = (u_2, v_2)$

$\Rightarrow (u_1 = u_2) \Rightarrow (v_1 \neq v_2)$

$\wedge (u_1 = v_2) \Rightarrow (v_1 \neq u_2)$

- Two edges e and f are <u>incident</u>

$$\exists u \quad e = (u,v) \wedge f = (u,w)$$

- $d(v) = |\{u \mid (u,v) \in E\}|$ = Degree (of vertex v)

$\equiv$ The number of vertices adjacent to a given vertex v.

$$\sum_{v \in V} d(v) = 2|E| = 2m$$

- Average degree of the graph

$$\bar{d} = \frac{\sum_{v \in V} d(v)}{|V|} = \frac{2m}{n}$$

Density = Ratio of the number of edges to the number of possible total.

$$= \frac{|E|}{\binom{n}{2}} = \frac{2m}{n(n-1)} = \frac{\sum d(v)}{n(n-1)} = \frac{\bar{d}}{n-1}$$

Density = 1 $\Rightarrow$ Graph is $\boxed{\text{complete}}$

$\bar{d} = n-1$

$\because \forall_v \; d(v) \leq n-1$

$\Rightarrow \forall_v \; d(v) = n-1$

A graph is complete if all of its vertices are adjacent to all others

$\Rightarrow$ D.O.S = 1

(Degrees of Separation).

Random Networks: