Social Networks

Lecture #15

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Auction & Honesty.

(Used by Google & Ebay etc.)

Second Price Auction:

(With Complete Information)

→ These assumptions can be relaxed!

1) Players.: $I = \{1, 2, 3, \ldots, n\}$

2) Strategies: $S_i = B_i ; \{b_1, b_2, \ldots, b_n\}$

Bids — Strategy Profile

$B_i = [0, \infty] = \mathbb{R}_+$ $\quad \in \mathbb{R}^n_+$

Players simultaneously submit bids b_i, $i \in I$.

$$S = \prod_{i \in I} S_i = B^n.$$

OBJECT: Outcome.

An indivisible object to be assigned to a single player $i \in I$.

$\forall_{i \in I} \; \exists!_{v_i \in \mathbb{R}}$ v_i = Player i's "private" valuation of the object.

WLOG, assume

$$v_1 > v_2 > \cdots > v_n.$$

Winner = Player $i \in I$. (to be determined)

Winner pays $\alpha(b) = \alpha(b_1, \ldots, b_n)$

Utility Function: $u_i : B \to \mathbb{R}$

$$: \langle b_1, b_2, \ldots, b_n \rangle \mapsto \begin{cases} v_i - \alpha(b) & i = \text{winner} \\ 0 & \text{o.w.} \end{cases}$$

SECOND PRICE AUCTION.

1) Players simultaneously submit bids, $b_i, i \in I$

2) The object is assigned to the $\boxed{\text{highest bidder}}$ (with random tie breaking)

3) The winner pays the $\boxed{\text{second highest bid}}$

utility Function.

$$u_i : B \to \mathbb{R}$$

$$: \langle b_1, b_2, \ldots, b_n \rangle \mapsto \begin{cases} v_i - b_j & \begin{cases} i = \text{highest bidder} \\ j = 2^{nd} \text{ highest bidder} \end{cases} \\ 0 & \text{o.w.} \end{cases}$$

Nash Equilibrium:

$$b^* = \langle b_1^*, b_2^*, \ldots, b_n^* \rangle \text{ s.t.}$$

$$\forall_{i \in I} \; \forall b_i \in \mathbb{R}_+$$

$$u_i(\langle b_i^*, b_{-i}^* \rangle) \geq u_i(\langle b_i, b_{-i}^* \rangle)$$

Complete Information Version: of Vickery / 2nd Price Auction.

Assume: Everyone knows everyone else's valuations:

$$v = \{v_1, v_2, \ldots, v_n\}$$

$\boxed{\text{THM}}$

The game has a truthful Nash Eq. $\boxed{\text{HONEST}}$

$$\langle v_1, v_2, \ldots, v_n \rangle = b^* = \text{Nash. Eq.}$$

Proof:
Player 1 has no incentive to deviate since

a) If $b_1 > v_2$ it has no effect on $u_1(b) = v_1 - v_2$.

b) If $b_1 < v_2$, it decreases his payoff $\because u_1(b) = 0 \leq v_1 - v_2$. (Rationality).

Player $j \neq 1$ has no incentive to deviate.

a) If $b_j > v_1$, it decreases his payoff

$$u_j(b) = v_j - b_j < v_j - v_1.$$

b) If $b_j < v_1$, it has no effect on $u_j(b) = 0$.

$\Rightarrow$ NO PLAYER HAS ANY INCENTIVE TO DEVIATE. $\square$

INCOMPLETE INFORMATION CASE:

- Two additional Nash Eq.

$$b'^* = \langle v_1, 0, 0, \ldots, 0 \rangle$$
$$b''^* = \langle v_2, v_1, 0, \ldots 0 \rangle$$

- Honest bidding $\Rightarrow$ Weakly Dominant Nash Eq.