**Social*Networks**
*QUIZ #3*
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Q1. [5 ] On a triangle, each corner has an ant. Each ant starts to move on an edge towards another corner, chosen at random. What is the chance that none of the ants collide (but they all visit all three corners)?

SOLN.1 *Out of a total of eight possibilities (*$3$ *ants with* $2$ *choices each), exactly two result in no collisions, i.e., all three ants go clockwise or anti clockwise. Hence , the probability of a non-colliding ant-movement is* $1/4$.

Q2. [5 ] The triangle can be thought of as a cyclic-graph with three-vertices: $C_3$. Define a generalization, $C_n$ ($n \geq 3$), as a graph with $n$ vertices: $V = \{v_i | 0 \leq i < n\}$ and edges $E = \{(v_i, v_{i+1 (\mod n)}) | 0 \leq i < n\}$. Each vertex has an ant; each ant starts to move on an edge towards another vertex, chosen at random. What is the chance that none of the ants will collide (but visit all $n$ vertices)?

SOLN.2 *Out of a total of* $2^n$ *possibilities (n ants with 2 choices each), exactly two result in no collisions , i.e., all n ants go clockwise or anti clockwise. Hence , the probability of a non-colliding ant-movement is* $2^{-(n-1)}$.

Q3. [10 ] The triangle can be thought of as a complete-graph with three-vertices: $K_3$. Define a generalization, $K_n$ ($n \geq 3$), as a graph with $n$ vertices: $V = \{v_i | 0 \leq i < n\}$ and edges $E = \{(v_i, v_j) | 0 \leq i \neq j < n\}$. Each vertex has an ant; each ant starts to move on an edge towards another vertex, chosen at random. What is the chance that none of the ants will collide (but visit all $n$ vertices)?

SOLN.3 *Out of a total of* $(n-1)^n$ *possibilities (n ants with* $(n-1)$ *choices each), exactly* $(n-1)!$ *result in no collisions (while visiting all vertices) , i.e., all n ants go on one of the Hamiltonian cycles (there are as many of those as the permutations of the vertices that keeps* $v_0$ *fixed). Hence , the probability of a non-colliding ant-movement is* $(n-2)!/(n-1)^{n-1}$*. Note that if we relax the condition (just avoid collision), then the problem is harder. You will have to count all the* cycle covers *in addition to the Hamiltonian cycles.*