

**\*LOGIC\***

HW #3

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19 November 2013 (due in 2 weeks)

Q1. [10 ] Let  $K \neq \emptyset$  be a chain of theories in  $\mathcal{L}$ , i.e.,  $T \subseteq T'$  or  $T' \subseteq T$ , for all  $T, T' \in K$ . Show that  $\cup K$  is a theory that is consistent iff all  $T \in K$  are consistent.

Q2. [10 ] Define a *pairing function*:  $\varphi : \mathbb{N}^2 \rightarrow \mathbb{N}$  as  $\varphi(a, b) = (\sum_{i \leq a+b} i) + a$ . Show diagrammatically how it enumerates all pairs  $(a, b) \in \mathbb{N}^2$ . Give a definition of  $\varphi$  by means of a bounded  $\mu$ -operation. Show that  $\varphi$  is a bijective primitive recursive function.