LOGIC
QUIZ #3
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Once again, we return to the Island of Knights and Knaves (IKK) along with our Anthropologist. In this island, those called *knights* always tell the truth and *knaves* always lie. Furthermore, each inhabitant is either a knight or a knave.

Q1. [15] Rumor has it that there is gold buried in the island. You wish to find out from a native of the island whether the rumor is true, but the native is unwilling to tell you whether he is a knight or a knave. He has no objection to answering one question by saying *yes* or *no*. What question would you ask?

Soln1. *Just ask the question (Q:) Are you Knight iff (S:) there is gold on the island. Let the answer you get is A*

Native	S	Q	A
knight	T	T	T
knight	F	F	F
knave	T	F	T
knave	F	T	F

Thus

$$A \Leftrightarrow S$$

Equivalently you could have asked "Is it true that one of the two alternatives holds?

- *You are knave amd there is no gold on the island.*
- *You are a knight and there is gold on this island."*

Or just, "Are you the type who could claim that there is gold on this island?"

Q2. [5] Using *induction on the construction of a Boolean formula*, define the size of a formula $Sz\ \Phi$

Soln2. *Two cases to consider*

(o) $Sz\ \pi = 1$, *for all prime formulas* $\pi \in \mathcal{F}$

(s) $Sz\ \neg\alpha = 1 + Sz\ \alpha$; $Sz(\alpha\ o\ \beta) = 1 + Sz\ \alpha + Sz\ \beta$, *for all compound formulas* $\alpha, \beta \in \mathcal{F}$, $o \in \{\wedge, \vee\}$.