LOGIC
QUIZ #6
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Once again, we visit the Island of Knights and Knaves along with our Antropologist. In these islands, those called *knights* always tell the truth and *knaves* always lie. Furthermore, each inhabitant is either a knight or a knave.

Q1. [15] On another of these islands, there are seven comedians, who have agreed to do one-night standup gigs at two of the five hotels during a three-day festival, but each of them is available for only two of those days. The editor of Knight Times published the following schedule:

- Tomlin will do Aladdin and Caesars on days 1 and 2;
- Unwin will do Bellagio and Excalibur on days 1 and 2;
- Vegas will do Desert and Exaclaibur on days 2 and 3;
- Williams will do Aladdin and Desert in days 1 and 3;
- Xie will do Caesars and Exacalibur on days 1 and 3;
- Yankovic will do Bellagio and Desrt on days 2 and 3;
- Zany will do Bellagio and Caesars on days 1 and 2.

Note that there is a bit of an ambiguity about the exact schedule; for instance Tomlin may do Aladdin first and then Caesars (respectively on days 1 and 2), or in the opposite order – Caesars first and then Aladdin. However, it is believed that the editor of Knight Times is actually a knave; do you agree?

Soln1. *Yes, the editor is a knave, because the schedule leads to a contradiction. First encode each schedule by a Boolean variable: t will mean that Tomlin will first do Aladdin* $[A1]$ *and then Caesars* $[C2]$*, while* $\bar{t}$ *will mean the opposite order* $[C1]$ *followed by* $[A1]$*. Thus:*

$$\begin{array}{lll}
\neg(t \wedge w)[A1] & \neg(y \wedge \bar{z})[B2] & \neg(t \wedge z)[C2] \\
\neg(w \wedge y)[D3] & \neg(u \wedge z)[B1] & \neg(\bar{t} \wedge x)[C1] \\
\neg(v \wedge \bar{y})[D2] & \neg(\bar{u} \wedge \bar{x})[E1] & \neg(\bar{u} \wedge y)[B2] \\
\neg(\bar{t} \wedge \bar{z})[C1] & \neg(\bar{v} \wedge w)[D3] & \neg(u \wedge \bar{v})[E2] \\
\neg(\bar{u} \wedge \bar{z})[B2] & \neg(x \wedge \bar{z})[C1] & \neg(\bar{v} \wedge y)[D3] \\
\neg(v \wedge x)[E3] & &
\end{array} \qquad (1)$$

Each constraint is a Krom clause, giving rise to the following 2-SAT problem:

$$\begin{aligned}(\bar{t} \vee \bar{w}) \wedge (\bar{u} \vee \bar{z}) \wedge (u \vee \bar{y}) \wedge (u \vee z) \wedge (\bar{y} \vee z) \\ \wedge (t \vee \bar{x}) \wedge (t \vee z) \wedge (\bar{x} \vee z) \wedge (\bar{t} \vee \bar{z}) \wedge (\bar{v} \vee y) \wedge (v \vee \bar{w}) \\ \wedge (v \vee \bar{y}) \wedge (\bar{w} \vee \bar{y}) \wedge (u \vee x) \wedge (\bar{u} \vee v) \wedge (\bar{v} \vee \bar{x}) \qquad (2)\end{aligned}$$

There is a vicious cycle in the resulting Krom graph:

$$u \Rightarrow \bar{z} \Rightarrow \bar{y} \Rightarrow \bar{v} \Rightarrow \bar{u} \Rightarrow z \Rightarrow \bar{t} \Rightarrow \bar{x} \Rightarrow u. \qquad (3)$$