V22.0453-001: Honors Theory of Computation

Problem Set 2

All problems are worth 10 points. Due on Oct 21, 2010.

Problem 1

Prove that the following languages are not regular:

1. $\{0^n 1^m 0^n \mid n \geq 0\}$
2. $\{w \mid w \text{ is not a palindrome}\}$

Problem 2

Consider a new kind of finite automaton called an All-Paths-NFA. An All-Paths-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if *every* possible computation of M on x ends in a state from F. Note, in contrast, that an ordinary NFA accepts a string if *some* computation ends in an accept state. Prove that All-Paths-NFAs recognize the class of regular languages.

Problem 3

If A is a language, let $A_{-\frac{1}{2}}$ be the set of all first halves of strings in A so that

$$A_{-\frac{1}{2}} = \{x \mid \text{for some } y, |x| = |y|, \text{and } xy \in A\}$$

Show that if A is regular, so is $A_{-\frac{1}{2}}$.

Problem 4

Give context-free grammars that generate the following languages. Also give informal description of the PDAs accepting these languages. The alphabet is $\{0, 1\}$.

1. $\{w \mid \text{length of } w \text{ is odd}\}$
2. $\{w \mid w \text{ contains more 1s than 0s}\}$
3. $\{w \mid w \text{ is a palindrome}\}$

Problem 5

For a language A, let SUFFIX(A) denote the set of all suffixes of strings in A, i.e.

$$\text{SUFFIX}(A) = \{v \mid uv \in A \text{ for some string } u\}$$

Show that if A is a context-free language, so is SUFFIX(A).

Problem 6

Use the pumping lemma to show that the following languages are not context free:

1. $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$
2. $\{0^i 1^j \mid i \geq 1, j \geq 1, i = jk \text{ for some integer } k\}$