V22.0453-001: Theory of Computation

Problem Set 1

For this problem set, the alphabet is $\Sigma = \{0, 1\}$ unless otherwise specified. All problems are worth 10 points.

Problem 1

Draw diagrams of DFAs recognizing the following languages.

1. $\{w : w$ begins with a 0 and ends with a 1$\}$.
2. $\{w : w$ contains at least two 1's$\}$.
3. $\{w : w$ has an even number of 0's and an odd number of 1's$\}$.

Problem 2

Draw diagrams of NFAs recognizing the following languages.

1. $\{w : w$ contains the substring 0110$\}$.
2. $\{w : w$ starts with 0 and has even length, or starts with 1 and has odd length$\}$.
3. The star closure of $\{w : w$ is any string but 11 and 111$\}$.

Problem 3

Exercise 1.12 on page 85 of Sipser (In the new edition of the book, this appears as 1.16 on page 86).
Note: You can use the procedure to convert an NFA to an equivalent DFA that I described in class; it is a bit different from the one in the book.

Problem 4

Give a regular expression for each of the following languages.

1. $\{w :$ The length of w is a multiple of 3$\}$.
2. $\{w : w$ either starts with 01 or ends with 10$\}$.
3. $\{w : w$ does not contain the substring 001$\}$.

Problem 5

Exercise 1.16 on page 86 of Sipser (In the new edition of the book, this appears as 1.21 on page 86).

Problem 6

The procedure for converting an NFA to an equivalent DFA given in class yields an exponential blowup in the number of states. That is, if the original NFA has n states, then the resulting DFA has 2^n states. In this problem, you will show that such an exponential blowup is necessary in the worst case.

Define $L_n = \{w : \text{The } n\text{th symbol from the right is } 1\}$.

1. Give an NFA with $n + 1$ states that recognizes L_n.
2. Prove that any DFA with fewer than 2^n states cannot recognize L_n.

   Hint: Let M be any DFA with fewer than 2^n states. Start by showing that there exist two different strings, $x \neq y$, $|x| = |y| = n$, that drive M to the same state (by the Pigeon-Hole Principle). Then argue that the strings xz and yz, for any string z, must also drive the DFA to the same state.