CSCI-UA.0453-001: Theory of Computation

Problem Set 2

All problems are worth 10 points.

Collaboration is allowed, but you must write your own solutions. Write the names of your collaborators (and your own!).

Unless stated otherwise, you must show all intermediate steps and give proper justification or proof.

Problem 1

Prove that the following languages are not regular:

1. $\{0^n 1^m 0^n \mid n \geq 0\}$
2. $\{w \mid w \text{ is not a palindrome}\}$

Problem 2

Consider a new kind of finite automaton called an All-Paths-NFA. An All-Paths-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if *every* possible computation of M on x ends in a state from F. Note, in contrast, that an ordinary NFA accepts a string if *some* computation ends in an accept state. Prove that All-Paths-NFAs recognize the class of regular languages.

Problem 3

If A is a language, let $A_{-\frac{1}{2}}$ be the set of all first halves of strings in A so that

$$A_{-\frac{1}{2}} = \{x \mid \text{for some } y, |x| = |y|, \text{and } xy \in A\}$$

Show that if A is regular, so is $A_{-\frac{1}{2}}$.

Problem 4

Give context-free grammars that generate the following languages. Also give informal description of the PDAs accepting these languages. The alphabet is $\{0, 1\}$.

1. $\{w \mid \text{length of } w \text{ is odd}\}$
2. $\{w \mid w \text{ contains more 1s than 0s}\}$
3. $\{w \mid w \text{ is a palindrome}\}$

Problem 5

For a language A, let SUFFIX(A) denote the set of all suffixes of strings in A, i.e.

$$\text{SUFFIX}(A) = \{v \mid uv \in A \text{ for some string } u\}$$

Show that if A is a context-free language, so is SUFFIX(A).

Problem 6

Use the pumping lemma to show that the following languages are not context free:

1. $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$
2. $\{0^i 1^j \mid i \geq 1, j \geq 1, i = jk \text{ for some integer } k\}$