Problem M/A 3 from Puzzle Corner – Technology Review, 2021
Burgess H Rhodes, XVIII, 1960

The Problem.

"Suppose a billiard ball is hit straight from the corner of an $a \times b$ rectangular billiard table at an angle of 60° as shown. How far below the left bumper will the ball first strike the left cushion on its return?"

Solution. Insert the billiard table, with left cushion width a and left bumper length b, into the top left of the reference grids shown below. The billiard ball, hit from upper-left corner H, travels a path which in the reference grids is a straight line, impacting the right side of the grid at point R'. If the grid is folded, first along the vertical midline M and then in "accordion" fashion back upon the billiard table, the straight line will trace the actual path of the ball to the point R of first return on the left cushion.

The required distance D below the left bumper to R is the distance on the right side of the reference grid between R' and the nearest H. Distance L on the right side of the reference grid from the top H down to R' is $L = 2b \cdot \tan 30°$. Let $K = \left\lfloor \frac{L}{a} \right\rfloor$. Then

$$D = \begin{cases} L - Ka & \text{if } K \text{ is even, and} \\ (K+1)a - L & \text{if } K \text{ is odd.} \end{cases}$$

In terms of the table dimensions: for $m = 0, 1, 2, \ldots,$

$$D = \begin{cases} \frac{2b}{\sqrt{3}} - \left\lfloor \frac{2b}{a} \right\rfloor a & \text{if } m\sqrt{3} \le \frac{b}{a} < (m+\frac{1}{2})\sqrt{3}, \text{ and} \\ \left(\left\lfloor \frac{2b}{a} \right\rfloor + 1 \right) a - \frac{2b}{\sqrt{3}} & \text{if } (m+\frac{1}{2})\sqrt{3} \le \frac{b}{a} < (m+1)\sqrt{3}. \end{cases}$$

Illustration: $K = 2$, **Even**

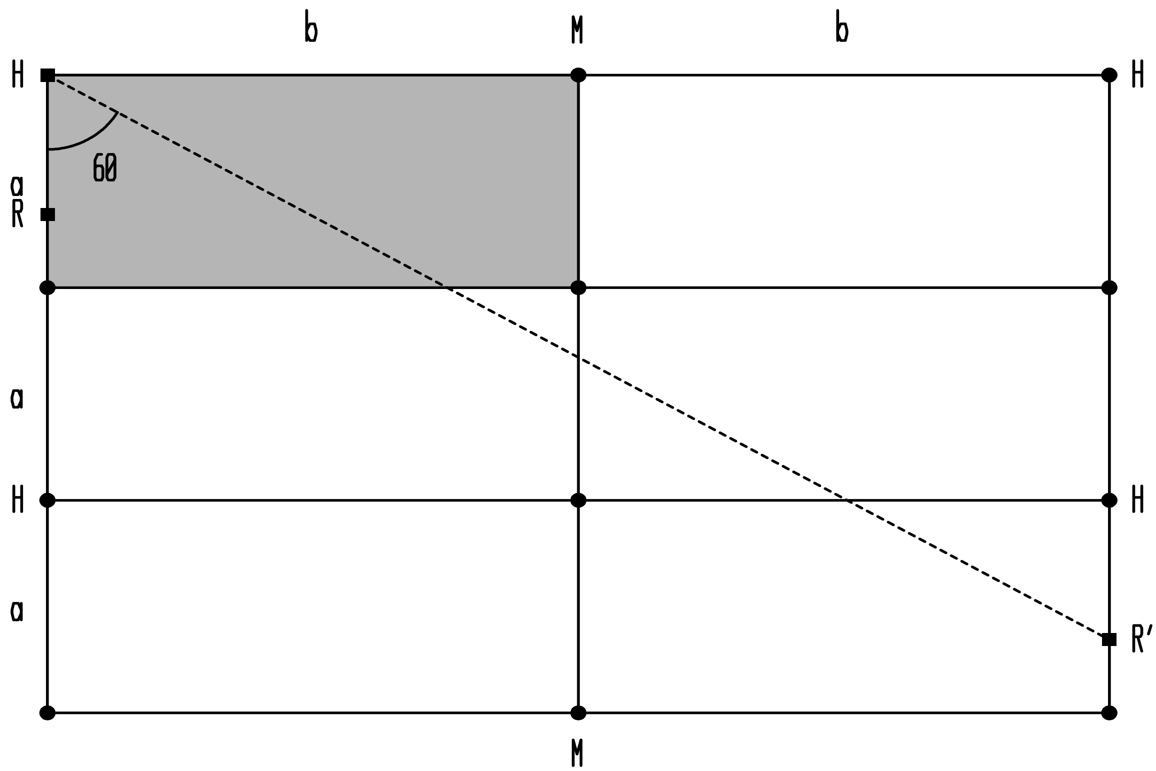

Illustration: $K = 3$, Odd