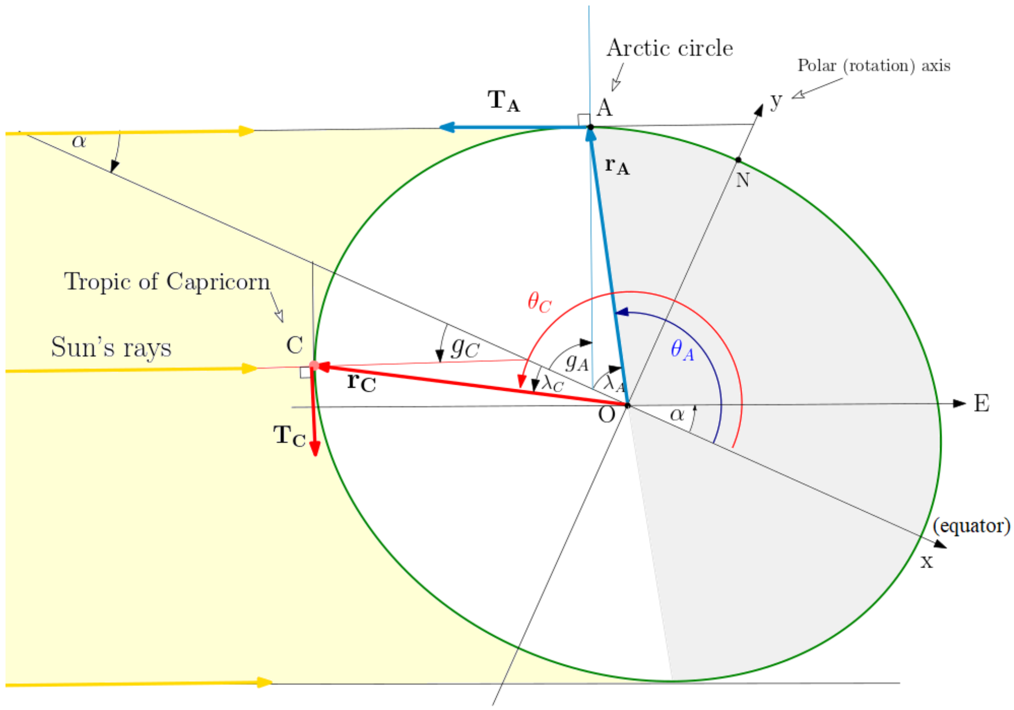

Problem M/A3. The earth (ellipsoidal shape exaggerated) at the December solstice. **E** is a line from the center of the sun through the center of the earth (the ecliptic plane). The equator of earth (the **x**-axis) is tilted by angle α =23.44 ° below the ecliptic. **A** marks the arctic circle, where light rays from the center of the sun are tangent to the earth's surface, making the *center* of the noon-day sun just barely visible there. At the same time, **C** marks the Tropic of Capricorn, where the center of the sun is directly overhead. (The earth-sun distance is so large that light rays from the center of the solar surface may be treated as parallel lines by the time they reach earth.) Points on the ellipsoidal surface of the earth are the vectors $r(\theta) = a\cos\theta\,\hat{i} + b\sin\theta\,\hat{j}$, where θ is the angle measured counter clockwise from the *x*-axis and *a* = 1 and *b* = .99665.

<u>**Geodetic latitudes**</u>: The geodetic latitude angles for **A** and **C** are labeled g_A and g_C , and right triangles in the diagram show clearly that, for the arctic circle, $g_A = 90° - \alpha = 66.56°$ (north) while, for the tropic of Capricorn, $g_C = \alpha = 23.44°$ (south)

<u>**Arctic Circle**</u>: The vector to ***A*** is $r_A = a\cos\theta_A\,\hat{i} + b\sin\theta_A\,\hat{j}$ and a vector tangent to the earth at ***A*** is $T_A = \frac{d\,r(\theta_A)}{d\theta} = -a\sin\theta_A\,\hat{i} + b\cos\theta_A\,\hat{j}$. A unit vector in the direction of the ecliptic line is $\hat{E} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$. At the arctic circle, $\mathbf{T}_A$ is parallel to $\hat{E}$, which means that

$\theta_A \sin\alpha - ¿\, b\cos\theta_A\cos\alpha$
$-a\sin ¿\,\hat{k}$, i.e. $\tan\theta_A = \frac{-b}{a}\cot\alpha$, so the *geocentric* latitude is
$T_A \times \hat{E} = 0 = ¿$

$\lambda_A = 180° - \theta_A = 66.49°$ (north).

Tropic of Capricorn: The vector from the center of the earth to ***C*** is $r_C = a\cos\theta_C \hat{i} + b\sin\theta_C \hat{j}$ and a vector tangent to **C** is $T_C = \frac{dr(\theta_C)}{d\theta} = -a\sin\theta_C \hat{i} + b\cos\theta_C \hat{j}$. Thus, the condition that $\mathbf{T}_C$ be perpendicular to the $\hat{E}$ can be written as $T_C \cdot \hat{E} = 0 = -a\sin\theta_C \cos\alpha + b\cos\theta_C \sin\alpha$, i.e. $\tan\theta_C = \frac{b}{a}\tan\alpha$, so the *geocentric* latitude is $\lambda_C = 23.37°$ (south).