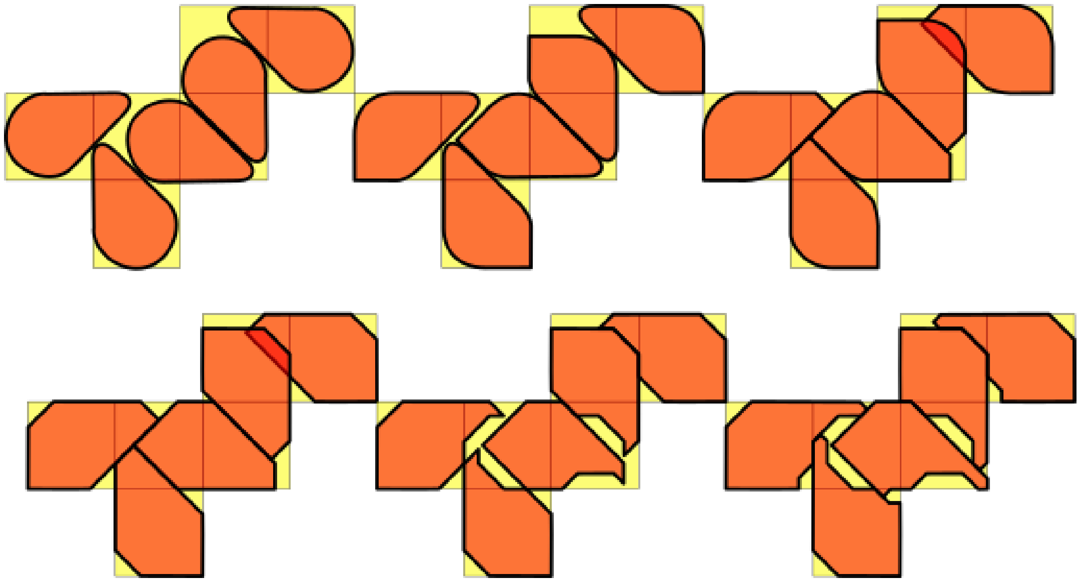
M/J – 3

Cover 88+% of a given arrangement of 6 squares with 5 congruent tiles (allowing mirrors and rotations)

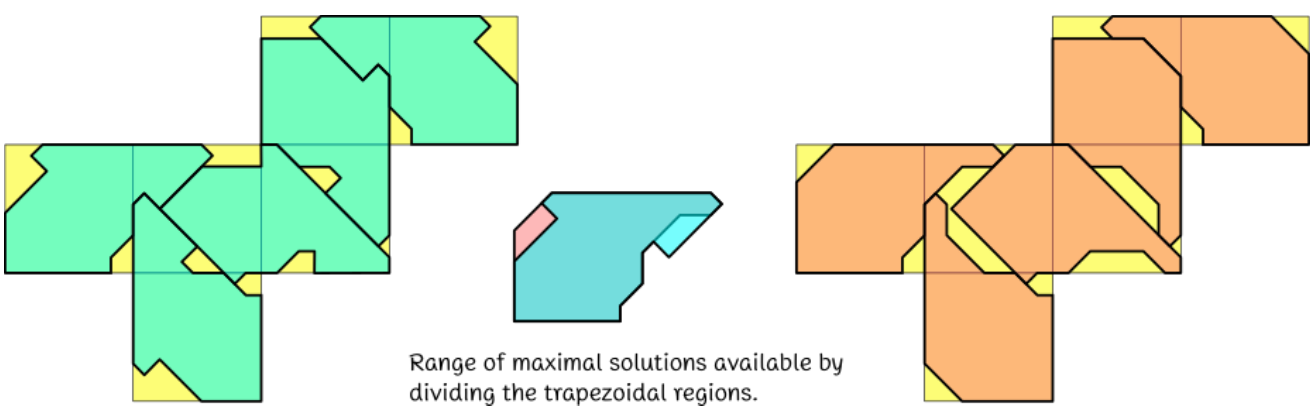
I used a practical approach of making and arranging clones in Inkscape, first to find a general arrangement, and then to iteratively adjust it in the following steps.



1. Decided on an overall arrangement by starting with ovals with 45-degree points that fit well in a common corner.
2. Filled out arc into free corner available to all tiles
3. Pushed the 45-degree edge as far out as possible (note: gains here outweigh the overlap losses from pushing one tile up. – The limit is when the base of the angled tile becomes locked to the edge of the square.)
4. Cleaned up the top hump on the angled piece.
5. Removed the overlap (there is some choice here of which piece to take it from, but I foresee taking it from the diagonal edge will allow picking up some extra space later.)
6. Picked up a small corner available to the vertically sliding piece, and extended the point on the angled piece into the corner.

The aha for the starting arrangement was in the 45-degree mating of the central (rotated) tile. Its mate has 2 possible orientations: rotated or reflected from its corner mate. I also tried the reflected version, but maxed out at 85% coverage, so I did not show it here.

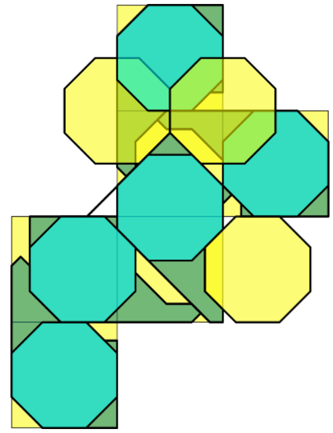
I mentioned some choice in the overlap in step 5. The tongue needs to remain, but the remaining trapezoidal area of the overlap is free to be divided as one wishes, so the maximal solution is not unique.



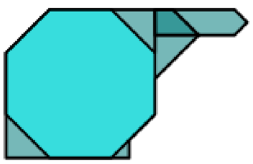
Range of maximal solutions available by dividing the trapezoidal regions.

The question remains as to what is the % coverage.

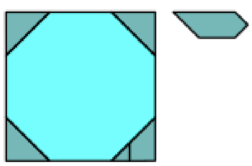
Notice that the 45-degree rotation together with the parallel sides of the squares gives rise to octagons that fit tightly inside the tiles. (And give another illustration of why this is maximal.)



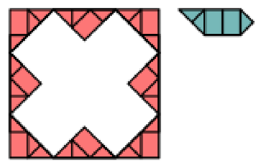
I divided the piece around the octagon that fits it. Then rearranged the parts to fill the square, which left an excess portion. I then sub-divided these parts with isosceles right triangles and rectangles based on the square root of two to find an expression of their relative areas and calculated the coverage from that.



Dissection of solution using octahedral basis.



Rearrangement to show excess beyond square area.



Area calculation:
Square edge = 4 + 3 * 2^(1/2)
Square area = 34 + 24 * 2^(1/2)
Extra area = 1.5 + 2 * 2^(1/2)
Total area of piece = 35.5 + 26 * 2^(1/2)
% coverage =
(5/6) * (35.5 + 26 * 2^(1/2)) / (34 + 24 * 2^(1/2)) =
88.6423749...%