2019 M/J 3

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Dear Dr. Gottlieb,

I do not know if this solution is optimal, but I enjoyed playing with this problem.

Subdivide the unit square into $3n$ by $3n$ squares for positive integer n. Remove a triangle of squares of side length $(n-1)$ from one corner of the unit square. From the opposite corner, remove a triangle of side length n, rotate it 180 degrees, and attach it to the edge the unit square so that it extends the staircase of squares it was detatched from. Add an $(n-1)$ by $(n+1)$ rectangle adjacent to both the rotated triange and unit square. Let $A(n)$ be the area of the tile: $A(n) = (19n^2 + n - 2)/(18n^2)$. For $3 \leq n \leq 122$, the area of the hexomino covered by five tiles is greater than 88%, with the maximum tile area being $A(4) = 17/16$, covering 85/96ths $= 88.541\overline{6}\%$ of the hexomino.

Similar, though slightly irregular, solutions exist where the unit square is subdivided $3n + 1$ and $3n + 2$ times per side, though from examining a handful of test cases, they appear to be worse than nearby solutions of the form $3n$.

Thank you for another entertaining puzzle corner,
Frank

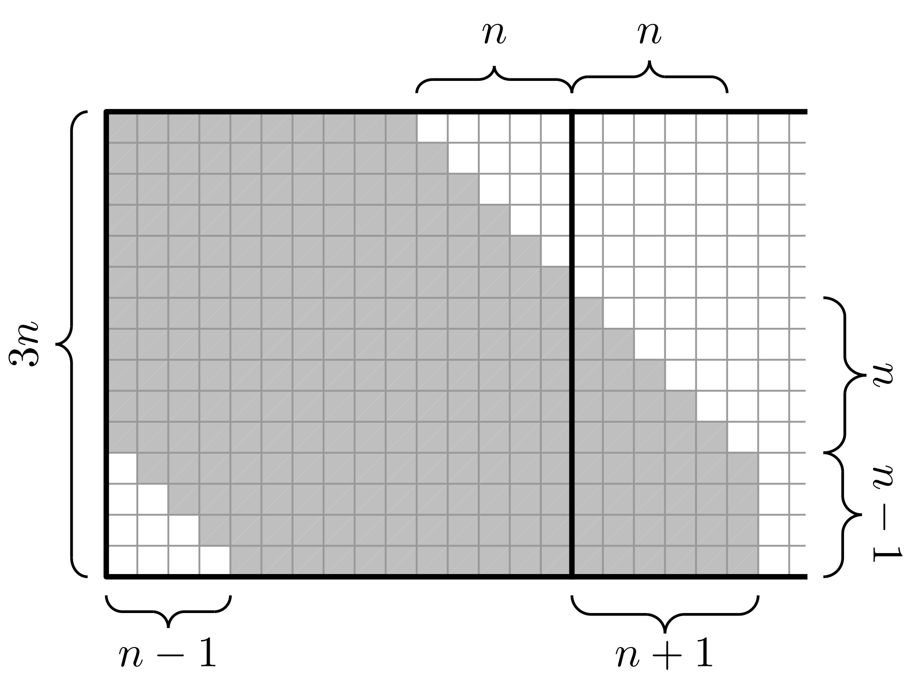

Figure 1: Construction of a single tile. The $n = 5$ case is shown.

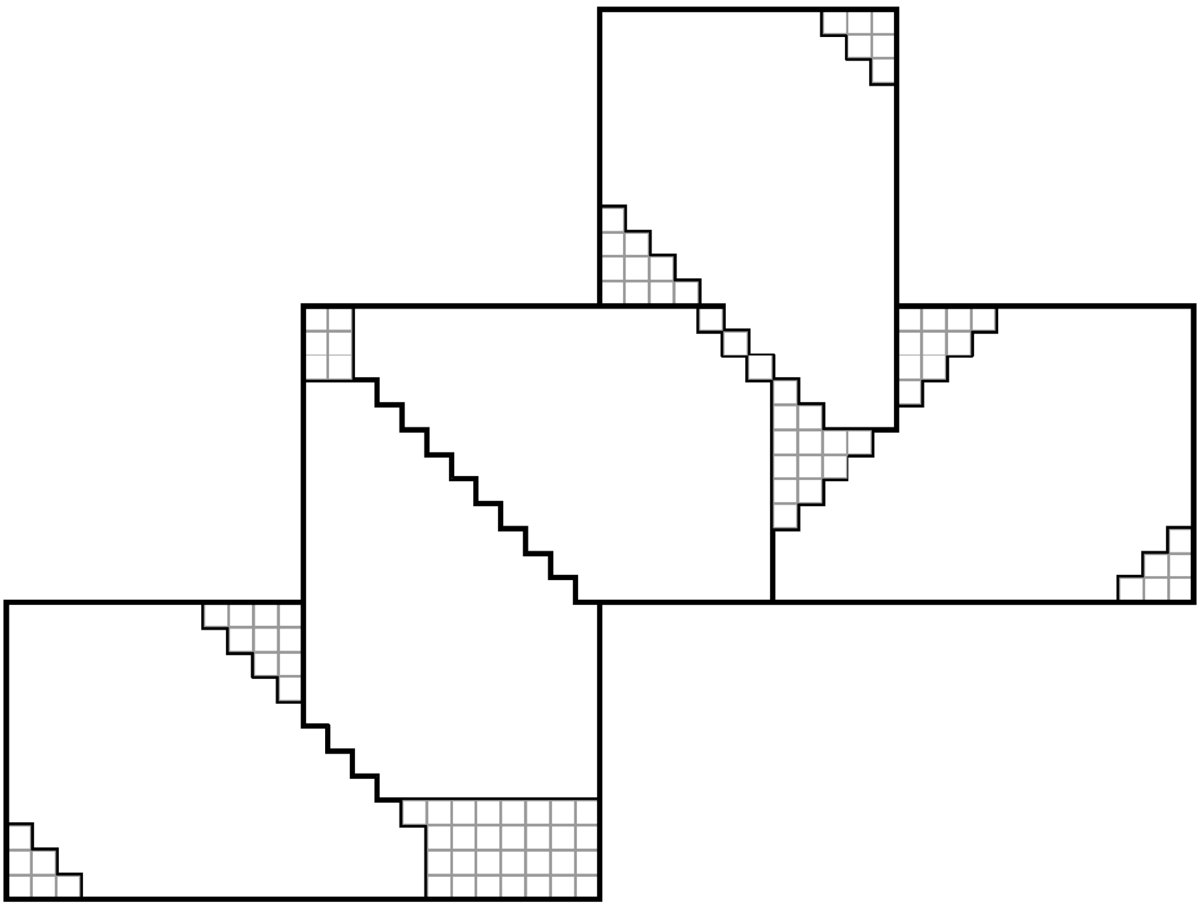

Figure 2: The $n = 4$ solution, which is the maximum coverage solution for this family of solutions.