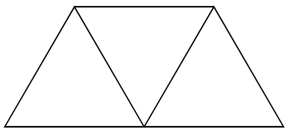
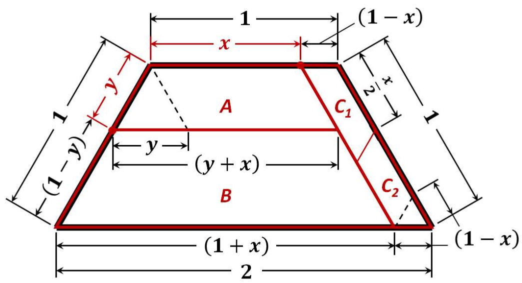
J/A 2. The late Dick Hess attributed the following problem to Bob Wainwright. A trapezoid comprising three equilateral triangles is to be divided into four similar areas of three different sizes (i.e., exactly two pieces are congruent).



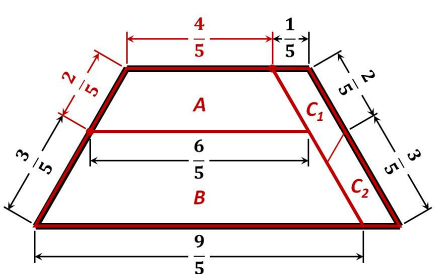
The triangle sides are assigned unit length. As diagrammed below, trapezoidal regions A, B, and C_1-C_2 (a congruent pair) may be formed by two lines, one parallel to the right side and one horizontal. Locations of the lines along the top and left side that create similar areas are unknown *a priori* and are designated x and y. These dimensions fully specify all the others.



Each trapezoid has base angles of $\pi/3$, hence the base width equals the top width plus the slant height. Similarity of A, B, C_1 and C_2 therefore requires only that the ratio of top width to slant height be matched. The two conditions $A \sim C$ and $A \sim B$ determine x and y as follows:

$$A \sim C: \quad \frac{x}{y} = \frac{x/2}{1-x} \qquad \therefore\ y = 2(1-x)$$

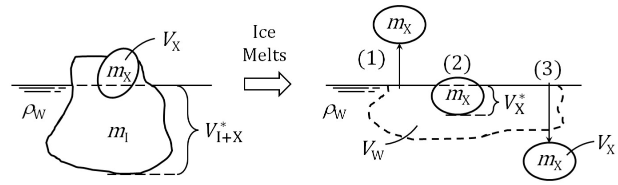
$$A \sim B: \quad \frac{x}{y} = \frac{y+x}{1-y} \quad \rightarrow \quad \frac{x}{2(1-x)} = \frac{2-x}{2x-1} \qquad \therefore\ x = \frac{4}{5}\ ,\ y = \frac{2}{5}$$



Corresponding sides of B, A, and C_1 or C_2 are in the ratio 3:2:1, thus the area ratio is 9:4:1. In all regions the top width is 2/3 times the base width and 2 times the slant height. Each internal trapezoid thereby comprises five equilateral triangles.

J/A 3. Spyros Kinnas offers three variants of the "floating ice problem". When ice floating on water melts, the water level is unchanged. He asks what happens to the water level when floating ice melts if: (1) the ice contains trapped pockets of air (of negligible weight); (2) a solid less dense than water (e.g., wood) resides in or atop the ice; and (3) a solid denser than water (e.g., steel) resides in or atop the ice.

A mass of ice m_I with an inclusion of dissimilar species X, shown on the left, floats with a volume V_{I+X}^* below the water level. The submerged volume supports the total mass via a buoyant force equal to the weight of displaced water. Hence $V_{I+X}^* = (m_I + m_X)/\rho_W$ where ρ_W is the density of water.



On melting (above right), the ice is converted to an equal mass of water having a volume $V_W = m_I/\rho_W$. Therefore:

$$V_{I+X}^* = V_W + (m_X/\rho_W)$$

This relationship is applied in turn to the three cases:

(1) Air has negligible mass and fully escapes the water. Hence $m_X \approx 0$, therefore $V_{I+X}^* = V_W$: the meltwater volume matches the original submerged volume exactly, as for ice alone. The water level is unchanged.

(2) A solid less dense than water stays afloat, displacing a volume of water V_X^* equivalent to its own weight. Then V_X^* equals (m_X/ρ_W), whereupon $V_{I+X}^* = V_W + V_X^*$: the portion of the pre-melt submerged volume that balanced the ice weight becomes an equal volume of meltwater, while the portion that supported the solid is occupied after melting by the solid alone. There is no change in water level.

(3) A solid denser than water sinks, displacing its own volume V_X of water rather than the larger V_X^* equivalent to its weight. Thus $V_{I+X}^* > V_W + V_X$: the solid displaces less water after the ice melts, therefore the water level drops.