Starting with the properties that

$$\begin{aligned} \min\{a_i,b_i\} &< a_{i+1} < \max\{a_i,b_i\} \\ \min\{a_i,b_i\} &< b_{i+1} < \max\{a_i,b_i\} \end{aligned}$$

We negate the second inequalty

$$-\max\{a_i,b_i\} < -b_{i+1} < -\min\{a_i,b_i\}$$

and then add it to the first to obtain

$$\min\{a_i,b_i\}-\max\{a_i,b_i\} < a_{i+1}-b_{i+1} < \max\{a_i,b_i\}-\min\{a_i,b_i\}$$

which is

$$-(\max\{a_i,b_i\}-\min\{a_i,b_i\}) < a_{i+1}-b_{i+1} < (\max\{a_i,b_i\}-\min\{a_i,b_i\})$$

This leads to

$$0 < |a_{i+1}-b_{i+1}| < |\max\{a_i,b_i\}-\min\{a_i,b_i\}|$$

(The absolute value on the right handside is unnecessary since the max is always larger than the min, but it will become useful in the next step).
$\max\{a_i,b_i\}-\min\{a_i,b_i\}$ can either be a_i-b_i or b_i-a_i . Inside the absolute value it will not matter. Consequently,

$$0 < |a_{i+1}-b_{i+1}| < |a_i-b_i|$$

Thus

$$\frac{|a_{i+1}-b_{i+1}|}{|a_i-b_i|} < 1$$

And the proof continues as in the posted solution.