Solution to J/A 2

When picking two cards, the probability of the sum being odd, P(Odd), equals the probability that one card will be odd and the other even. This can occur if the first card is odd and the second is even or if the first card is even and the second is odd. Because the cards are drawn with replacement, and all face cards are valued at ten, the probability that any drawn card is odd is 20/52 = (1/2 – 6/52) and the probability that any drawn card is even is 32/52 = (1/2 + 6/52).

Then for the sum:

$P(Odd) = (1/2 - x)(1/2 + x) + (1/2 + x)(1/2 - x) = 2 * (1/4 - x^2) = 1/2 - 2x^2$, where $x = 6/52$
$P(Even) = 1 - P(Odd) = 1/2 + 2x^2$, and therefore the more likely outcome is an even sum, with a probability of 0.5266.

If four cards are drawn and summed, this is equivalent to two trials of the initial experiment, with each trial resulting in odd or even sums as per the equations above, and the two trial sums then added together. Therefore, for four cards:

$$P(Even) = 1/2 + 2(2x^2)^2 = 1/2 + 2^3x^4 = 0.5014$$

Alternatively, consider that n cards have been drawn, and their sum is even with probability $P_n(Even)$. Then one additional card is drawn, which can be even or odd. The overall sum is even if the previous sum is even and the new card is even or if both are odd. Thus,

$P_{n+1}(Even) = P_n(Even)*(1/2 + x) + [1 - P_n(Even)]*(1/2 - x) = 1/2 - x + 2x*P_n(Even)$,
or
$P_{n+1}(Even) = 1/2 + 2x*[P_n(Even) - 1/2]$

Starting with $P_2(Even)$, as initially shown above, and extrapolating, we find:

$$P_n(Even) = 1/2 + 2^{(n-1)}x^n$$

This confirms the prior answer for $P_4(Even)$ and allows us to calculate:

$$P_{100}(Even) = 1/2 + 2^{99}(6/52)^{100} = 0.5 + (3/26)*(3/13)^{99}$$