

It took me a little while to understand the problem and see what it wasn't trivial—that we are talking about the probability of being ahead of another strategy, independently of by how much, not about expected values.

Let $S(c, n)$ be the stake of a player using constant c after n coin flips. We have

$$S(c, n + 1) = S(c, n) \times ((1 - c) + c \times [2 \text{ or } 0.4])$$

The second factor is either $1 + c$ or $1 - 0.6c$.

Assume first that n is a large even number. There are $n + 1$ outcomes, whose probabilities are determined by a binomial expansion. Let us ask what value of c maximizes the expected value of the most probable outcome, in which there are $n/2$ heads and tails. In that case, the new stake is

$$S(c, n) = S(c, 0) \times ((1 + c)(1 - 0.6c))^{(n/2)}$$

What value of c maximizes the value raised to the power? The value is $-0.6c^2 + 0.4c + 1$; its derivative is $-1.2c + 0.4$ and the latter is 0 when $c = 1/3$ (the second derivative is negative, so this is a maximum).

So $c_{\max} = 1/3$ wins for the outcome for equal numbers of heads and tails. For outcomes with fewer heads than tails, there are some number of $((1 + c)(1 - 0.6c))$ “paired” factors, and some number of excess $(1 - 0.6c)$ factors. For outcomes with more head than tails, we have excess $(1 + c)$ factors. The total probability of these sets of unequal outcomes are the same. If $c > c_{\max}$, then c_{\max} wins all the former cases; if $c < c_{\max}$, c_{\max} wins all the latter cases. Since c_{\max} wins the “middle” case, it wins in a majority of cases.

For the case where n is odd, we consider the two middle cases, which have one excess head or tail, and thus one unpaired $(1 + c)$ or $(1 - 0.6c)$ factor. For $c \neq c_{\max}$, the ratios $(1 + c)/(1 + c_{\max})$ or $(1 - 0.6c)/(1 - 0.6c_{\max})$ may exceed 1. However, since c_{\max} maximizes $(1 + c)(1 - 0.6c)$, the ratio of this for c_{\max} to any other value of c exceeds 1. And this is raised to the power $\lfloor n/2 \rfloor$. As $n \rightarrow \infty$, this will exceed the single-term ratio, so, as n gets large, c_{\max} wins for both middle terms. As before, c_{\max} will win for either the rest of the cases with more or fewer heads, so it wins overall.

Obviously, since the expected value of a trial is positive, $c = 1$ maximizes the expected value, but it does not lead to the most-likely to dominate strategy.