John Chandler writes that it is refreshing to have a search that can be conducted just by logic, rather than requiring an exhaustive examination of many cases. Indeed, Chandler's solution, a slightly modified version of which follows, avoids an exhaustive search in most cases.

If we require $AB \times C = A \times BC$, then, in particular, we must have the same units digit in both products, and so we know $B \times C - A \times C \equiv 0 \pmod{10}$, i.e., $(B - A) \times C \equiv 0 \pmod{10}$.

Since the digits can't be zero, that means either C is 5 and $B - A$ is even, or $|A - B|$ is 5 and C is even. The first case implies $(10A + B) \times 5 = A \times (10B + 5)$ or $B = 9A/(2A - 1)$. The only integer solutions occur when $2A - 1$ is a divisor of 9, and so the only solutions for A and B are 1,9; 2,6; and 5,5. The last is ruled out because the digits must be distinct, and the second is the example we started with. Thus, we have just one new example from this case: $19 \times 5 = 1 \times 95$.

The other case is actually two cases: $A = B + 5$ or $A = B - 5$. Given $A = B + 5$, we see that: $(11B + 50) \times C = (B + 5) \times (10B + C)$ or $B^2 + (5 - C) \times B - 4.5C = 0$. Thus $B = \dfrac{C - 5 \pm \sqrt{C^2 + 8C + 25}}{2}$. The argument of the square root can be rewritten as $(C + 4)^2 + 9$, which points to the familiar 3-4-5 pythagorean triple, and so the only value of C giving us an integer square root in this expression is $C = 0$, which is forbidden.

Given $A = B - 5$ we see that: $(11B - 50) \times C = (B - 5) \times (10B + C)$ or $B^2 + (-5 - C) \times B + 4.5C = 0$. Hence $B = \dfrac{C + 5 \pm \sqrt{C^2 - 8C + 25}}{2}$. Here, we get an integer when C is 8 or 4. $C = 8$ gives $B = 9$ or 2. Since we require A to be positive, the only solution here is $B = 9$, whence we have $49 \times 8 = 4 \times 98$. $C = 4$ gives $B = 6$ or 3. Again $A > 0$ eliminates the latter yielding only $16 \times 4 = 1 \times 64$.

The reasoning is very similar for finding cases of $A \times BCD = ABC \times D$, i.e., we see that either $|A - C| = 5$ and D is even, or $|A - C|$ is even and D is 5. In the latter case, we have $A \times (100B + 10C + 5) = (100A + 10B + C) \times 5$ or $10B + C = 99A/(2A - 1)$. Again, we must have 2A-1 be a divisor of the leading coefficient, this time 99, giving sets of A, B, C: 1,9,9; 2,6,6; 5,5,5; 6,5,4. Since the digits must all be distinct, these are all rejected.

Similarly, for the other two equalities, we get a class of possible solutions for $AB \times CD = A \times BCD$ with $|B - A|$ even and $D = 5$, and for $AB \times CD = ABC \times D$ with $|B - C|$ even and $D = 5$. These conditions lead to: $(10A + B) \times (10C + 5) = A \times (100B + 10C + 5)$ or $A = B \times \dfrac{2C + 1}{20B - 9 \times (2C + 1)}$ When $2C + 1$ is prime, it must divide either B or 20 to make A come out as an integer, and, if not prime, then it must at least have a common divisor with B, and we see that B must be greater then C to make A come out positive, so we are limited to B, C = 3,1; 6,1; 9,1; 3,2; 7,3; 9,7. Not all of these give an integer value for A, but we do find two: $13 \times 25 = 1 \times 325$ and $39 \times 75 = 3 \times 975$

$(10A + B) \times (10C + 5) = (100A + 10B + C) \times 5$ yields $10A + B = C/(2C - 9)$ Clearly, no one-digit value of C will work.

Things get more complicated in the other cases, where $D \neq 5$. Indeed, so complicated that I don't see any refreshing logic to home in on the possible solutions. Hence, I retreat to the exhaustive search and find that there is indeed another solution in this case: $27 \times 56 = 2 \times 756$.