

The first digit can be neither 0 nor 1 because these indicate the following is a long distance phone call. The second and third digits cannot both be 1 because the three digit sequence n\*11 are reserved (411, 911, etc.). Also numbers of the form 555-xxxx are reserved for use in entertainment and examples. So the total number of valid phone numbers is  $8 * 10^6 - 8 \times 10^4 - 1 \times 10^4 = 7910000$

How many valid phone numbers have the desired property?

If the first digit is 8, pick 3 out of the remaining 6 positions to have the same value as Jenny's.  $C(6, 3) = 6! / (3! \times (6 - 3)!) = 20$ . Each of the 3 remaining digits may be any value, except the value in the corresponding position in Jenny's number, i.e.,  $9^3 = 729$ . The total is  $20 \times 729 = 14580$ .

If the first digit is not 8, pick 4 out of the remaining 6 position to match Jenny's.  $C(6, 4) = 6! / (4! \times (6 - 4)!) = 15$  The first digit can't be 0, 1, or 8, but the other two can be any of 9 values, i.e.,  $7 * 9 * 9 = 567$ . The total is  $15 \times 567 = 8505$ .

But some of these are invalid. First exclude the 811-xxxx numbers. Exactly one of the last 4 digits is the non-matching one, giving  $4 \times 9 = 36$  phone numbers to exclude. Next exclude the n11-xxxx numbers. Because none of the first three digits match Jenny's, we know xxxx must be 5309. Therefore, there are 7 phone numbers to exclude. Finally, exclude 555-5309.

Thus the total number of valid phone numbers with the desired property is  $14580 + 8505 - 36 - 7 - 1 = 23041$  and the desired probability is  $23041 / 7910000$  or approximately 0.2913%.