May 27, 2014

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Three hundred sixty-five random people – How many birthdays?

First – A true story:

There was a rich man who was a big shot at his golf club. One day he shouted out to a large congregation in the clubhouse: "I'M GOING TO THROW A BIG, BIG BIRTHDAY PARTY FOR EVERYBODY!"

When asked, he explained: "For next year; starting on Jan 1, and continuing throughout the year, I will host a birthday party for all the people who have birthdays on that particular day."

He also explained: "This will just be for the first 365 people who sign-up; starting now."

365 people quickly signed-up.

The staff at the club asked him how many days they would have to provide the birthday cake and all the meals, etc. Surely, there couldn't be a birthday each day.

The big shot gathered all 365 men* who signed-up and led them out to the number 1 fairway. He asked them all to form a straight line; facing him, and to each hold up a placard with their birthday printed on it.

Then he told everyone: "Look to your right; and, if you see a placard with your birthday on it, step out of line and go onto the green." When everybody had done as they were told, the remaining people represented the number of separate birthday parties that needed to be staged.

So – the question is: On average, how many birthdays will there be (assuming that all birthdays are random, and we neglect leap-years)?

Let's do it the way the big shot did:

The number one man in line looks to his right, and sees 364 men holding up birthday numbers (like 4-1, etc.) What is the probability that he sees at least on number that matches his:

Probability of at least one match = (I – probability of no match)

Probability of not matching 364 men = $(364/365)^{364}$

So; the probability that number one sees a match (and therefore drops out):

Probability of at least one match = $(I - (364/365)^{364})$

Probability that number 2 sees a match = $I - (364/365)^{363}$

And so on:

Probability that nth person sees a match = $Pn = I - (364/365)^{(365-n)}$

The 364th guy* is the last to look:

Probability that number 364 sees a match = $I - (364/365)^{(1)} = 1/365$

Number 365 needn't do anything (just stand there with his placard.)

The probable number of "dropouts" is, therefore:

Number of drops = $\sum Pn$ (n = 1-364)

Running the calculations:

$\sum Pn$ = 134.09 (or 134 rounded)

So the probable number of birthdays:

Birthdays = 365 – 134 = 231

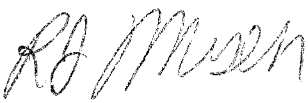
Now the head chef at the club says:

"Ok, there will be 231 birthday parties, on average, over the course of the year. I have heard that there is an average, and there is a standard deviation. I've been told that the total of average + 3 standard deviations is about the practical maximum that can be expected."

(I leave it to the interested student to determine the probably standard deviation in this situation.)

*Really – women don't join golf clubs, only men.

Sincerely,



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