2012 M/J 3 Addendum

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Previously we looked for decimal numbers for which the right cyclic permutation equals the double.

Additional solutions exist in bases other than decimal. For example the base 5 numbers 13_5, 102342_5 and 210234_5 are solutions. The base 9 solutions are 25736315_9, 15257363_9, 31525736_9, 36315257_9, 10467842_9, 21046784_9 and 42104678_9.

Solutions also exist for multiples other than two. In base b, for multiple $m : 2 \leq m < b$, as many as $b - m$ solutions exist (before self-concatenated solutions), one solution ending in each digit from m to $b - 1$. I do not know if all $b - m$ solutions exist for all b.

A familiar pattern emerges for multiple $(b - 1)$ in base b:

$(1\ 0\ 1\ 1\ 3)_4$

$(1\ 0\ 1\ 1\ 2\ 4\ 2\ 1\ 4)_5$

$(1\ 0\ 1\ 1\ 2\ 3\ 6\ 3\ 2\ 6\ 2\ 1\ 3\ 5\ 2\ 0\ 2\ 2\ 5\ 0\ 5\ 6\ 5\ 5\ 4\ 3\ 0\ 3\ 4\ 0\ 4\ 5\ 3\ 1\ 4\ 6\ 4\ 4\ 1\ 6)_7$

$(1\ 0\ 1\ 1\ 2\ 3\ 5\ 9\ ...\ 9)_{10}$

$(1\ 0\ 1\ 1\ 2\ 3\ 5\ 8\ 14\ ...\ 14)_{15}$

$(1\ 0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 22\ ...\ 22)_{23}$

$(1\ 0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 35\ ...\ 35)_{36}$

$(1\ 0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34\ 56\ ...\ 56)_{57}$

$(1\ 0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34\ 55\ 90\ ...\ 90)_{91}$

$(1\ 0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34\ 55\ 89\ 145\ ...\ 145)_{146}$

$(1\ 0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34\ 55\ 89\ 144\ 234\ ...\ 234)_{235}$ (24490 digits)

Let F_n be the n^{th} term in the Fibonacci sequence. Solutions for multiple $(b - 1)$ in base b such that $F_n + 2 \leq b < F_{n+1} + 2$ begin with the digit 1 followed by the first n terms of the Fibonacci sequence. Long division of $((b - 1)\ \ 1\ 0\ 1\ 1\ 2\ 3\ 5\ ...\ F_{k-3}\ ...)$ gives some insight: Assume that for $k \geq 2$, division of the k^{th} term from the left by $(b - 1)$ leaves a remainder of F_{k-2}. The next term in the long division is $(F_{k-2} * b + F_{k-3})/(b - 1) = F_{k-2} + (F_{k-2} + F_{k-3})/(b - 1) = F_{k-2}$ plus a remainder of F_{k-1}.