Re: the solution to problem N/D 3 – Technology Review, November/December issue 2011 (the time to fill a bowl while liquid evaporates)

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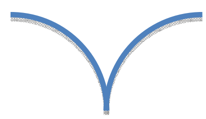
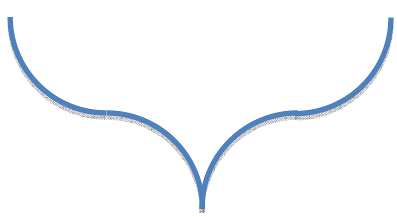
For this problem, it is not quite correct that any bowl shape will take an infinite amount of time to fill. For example, a bowl (a solid of revolution) with the "seagull" shape cross-section below—a profile composed of two 90° circular arcs—will fill in a finite amount of time. The intuitive reason for this relates to the fact that, in Burgess Rhodes' symbology, $S'(H_0) = \infty$, where $S' = dS/dh$, and H_0 is the value of h at which evaporation perfectly offsets filling ($H_0 = R$, the radius of the arcs below). (S is the surface area of the top liquid surface, and h is the height of filling.) More specifically, the intuition goes, since the surface area increases infinitely quickly (with respect to increasing h) as the height of the liquid reaches H_0, the evaporation also increases infinitely quickly; but if h is at all below H_0, the evaporation rate is simply no match for the filling rate, so h will continue to increase at a finite rate (until $h = H_0$).



I derived (but haven't double-checked) an analytic solution for the time to fill this seagull bowl, whose profile is described by $(x-R)^2 + y^2 = R^2$ for $0 \le x \le R$; and by $(x+R)^2 + y^2 = R^2$ for $-R \le x \le 0$ —or, equivalently but perhaps more succinctly, described by $(|x|-R)^2 + y^2 = R^2$ for $-R \le x \le R$.

My solution is: $T_0 = \frac{R}{e}\left(\frac{2\pi}{3\sqrt{3}} - 1\right) \approx 0.2092\frac{R}{e}$; perhaps some other reader can verify it!

I'm not exactly sure where Burgess Rhodes' analysis breaks down. He assumes that $S(h)$ is continuous, but the seagull cross-section can be made to be continuous basically everywhere (except at $h = 0$, but that's not an important exception for this problem) by extending the seagull wings, e.g. by using additional 90° circular arcs to make the cross-section below:



But presumably his analysis somehow breaks down under the condition $S'(H_0) = \infty$.