SOLUTION TO PROBLEM M/A3 (Technology Review, March/April 2011, page M62)

(Refer to attached figure)

Write the law of cosines for angle θ for triangles ABC and ADC

For $\triangle$ABC: $\left(r_2+\rho\right)^2=\left(r_1+r_2\right)^2+\left(r_1+\rho\right)^2-2\left(r_1+r_2\right)\left(r_1+\rho\right)\cos\theta$

For $\triangle$ADC: $\left(r_1+r_2-\rho\right)^2=r_2^2+\left(r_1+\rho\right)^2-2r_2\left(r_1+\rho\right)\cos\theta$

Equating $\cos\theta$ for these two triangles gives, after simplification,

$$\rho=\frac{r_1r_2\left(r_1+r_2\right)}{r_1^2+r_1r_2+r_2^2} \qquad (1)$$

From Pythagoras, $$\sin\theta=\frac{2\sqrt{r_1r_2\rho}\sqrt{r_1+r_2+\rho}}{\left(r_1+r_2\right)\left(r_1+\rho\right)} \qquad (2)$$

The distance from C to the baseline is given by $h=\left(r_1+\rho\right)\sin\theta$ (3)

Substituting equation (1) for ρ in equation (2) gives finally

$$h=\frac{2r_1r_2\left(r_1+r_2\right)}{r_1^2+r_1r_2+r_2^2}=2\rho$$

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