SPEED DEPARTMENT Problem – Technology Review, J/A 2011
Burgess H Rhodes, XVIII, 1960

The Problem.

> "George Bloom wishes to drill a hole clear through the center of a solid sphere. His flat-bottom drill is exactly six inches long, and he uses all of it in the drilling operation. How much material is left?"

Observation. Because a diameter for the drill is not given does not mean the diameter is irrelevant!

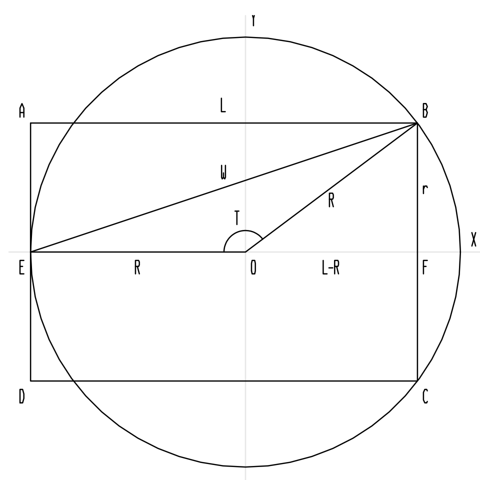
Basic Volume Formula. A circular hole of radius r is bored through a sphere of radius R on a diameter of the sphere. The volume remaining is determined by (use "cylindrical shells")

$$V(R,r) = 2\int_r^R 2\pi y\sqrt{R^2-y^2}dy = -2\int_{R^2-r^2}^0 \pi\sqrt{u}\,du = \frac{4\pi}{3}(R^2-r^2)^{3/2}.$$

A Flat-bottom Drill. Suppose the entire length L of a flat-bottom drill (such as a Forstner bit) with cutting radius r is used to bore the circular hole. At the moment the cutting rim of the flat bottom emerges through the surface of the sphere from the interior, the hole is complete. A spherical cap simply falls free.

In the following figure the rectangle with corners A, B, C, and D is the cross-section of the hole drilled in the sphere. The drill, which bores from left to right, begins cutting at point E on the sphere and proceeds a distance L. The cutting rim of the flat bottom has just touched the sphere's surface from the interior (points B and C). The segment on the right defined by chord B-C generates the spherical cap which falls off. The segment at the top above the line A-B generates the volume remaining after the hole is bored.

Cross Section – Sphere and Flat-bottom Bit
(Spin this figure around the X-axis to generate the full geometric figure.)



Equations which relate the various parts are

$$\text{angle } T = \pi - \tan^{-1}(r/(L-R))$$

and by the Pythagorean Theorem and law of cosines,

$$W^2 = L^2 + r^2 = R^2 + R^2 - 2 \cdot R \cdot R \cdot \cos T.$$

From these determine that the radius R of the sphere for which the flat-bottom drill of length L and radius r can just bore a diametric hole through the sphere is

$$R = \frac{L^2 + r^2}{2L}.$$

Volume Remaining. For the sphere with this radius R, the volume remaining is

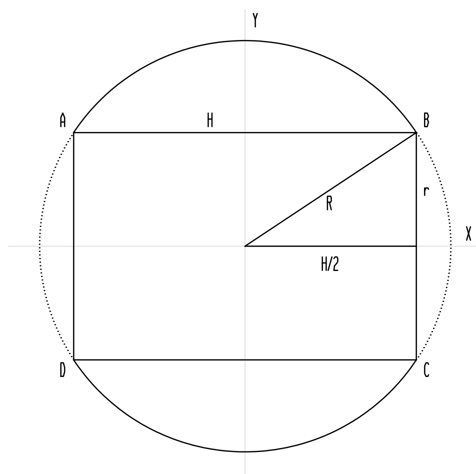
$$V(\frac{L^2 + r^2}{2L}, r) = \frac{4\pi}{3}\left(\frac{L^2 - r^2}{2L}\right)^3.$$

If $r = 0$ and $L = 6$ then the volume remaining is $\frac{4\pi}{3}3^3 = 36\pi$. Otherwise the volume remaining does depend upon both L and r, the dimensions of the drill.

An Alternate Look. Suppose after the hole is drilled the height of the hole is H. With radius of sphere equal to R, the required radius of the drill is $r = \sqrt{R^2 - (H/2)^2}$.

Cross Section – Sphere with Diametric Hole

(Spin this figure around the X-axis to generate the full geometric figure.)



In this figure the rectangle with corners A, B, C, and D is the cross-section of the hole, and the segment above the chord A-B generates the portion of the sphere remaining. The volume remaining is

$$V(R, \sqrt{R^2 - (H/2)^2}\,) = \frac{4\pi}{3}(R^2 - (R^2 - (H/2)^2))^{3/2} = \frac{4\pi}{3}(H/2)^3$$

which is independent of both the radius of the sphere R and the radius of the drill r. The volume remaining depends only upon the height H of the bored hole. If $H = 2R$ so that the drill has radius $r = 0$, the remaining volume is the entire volume $\frac{4\pi}{3}R^3$ of the sphere.