Given thirteen stacks of 4 coins each, knowing that one stack has identical counterfeit coins that weigh less than standard coins (by an amount not exceeding 5 grams), and knowing that good coins all weigh an integral number of grams, how can we determine the following in two weighings:

1) The weight of good coins
2) The stack with the counterfeit coins
3) The weight of the counterfeit coins?

The first part is easy: put more than 20 coins on the scale and weigh them. Divide the result by the number of coins. If the result (average weight of a coin) is not an integer, round it up to the next integer to find the weight each good coin. This works because even if all four counterfeit coins are in the group, they can only create a deficit of less than 20 grams, which when divided by a number larger than 20 must lower the apparent average weight of a coin by less than one gram.

To tackle the second and third questions, consider that in each weighing there can be zero, one, two, three, or four counterfeit coins – a total of five possibilities per weighing. This means there are a total of 25 possible outcomes for the two weighings. Let us define the “deficit” in each weighing as the difference between the ideal total weight of good coins and the actual total weight registered on the scale. I’ve shown in this table the ratio of the deficit from the first weighing divided by the deficit from the second weighing; examining the table we find that there are 13 unique ratios possible (and 12 duplicates).

		Counterfeit coins in First Weighing				
		0	**1**	**2**	**3**	**4**
Counterfeit coins in Second Weighing	**0**	NA	∞	∞	∞	∞
	1	0	1	2	3	4
	2	0	.5	1	1.5	2
	3	0	.3333	.6666	1	1.3333
	4	0	.25	.5	.75	1

Therefore, we can arrange coins from the different stacks in such a way that each stack can contribute deficits according to one of the unique ratios in the table. Then it becomes a simple matter to match up the deficit ratio we measured to the stack that caused it, and also a simple matter to calculate the missing weight in the counterfeit coins. In my solution, there will be 26 coins in each weighing. (You could choose to use different entries in the above matrix and have more than 26 coins in each weighing, even to the point of having a different number of coins in the second weighing compared to the first, but the useful ratios will all be from the same list.)

Notice that we do need to keep track of which coin came from which stack, because we're going to put some of the very same coins in both the first and second weighings. For example, in my solution below you can't just dump all four coins from stack 13 into an undifferentiated pile of coins in the first weighing because you need to put three of these same coins into the second weighing.

Stack	No in 1st weighing	No in 2nd weighing	Ratio of first deficit divided by second deficit
1	0	1	0
2	1	0	∞
3	1	1	1
4	1	2	0.5
5	1	3	0.3333
6	1	4	0.25
7	2	1	2
8	2	3	0.6666
9	3	1	3
10	3	2	1.5
11	3	4	0.75
12	4	1	4
13	4	3	1.3333
TOTAL	26	26	