## J/F Technology Review

Scott Howlett notes that the solution on the website assumes that the counterfeit coins are always lighter than the true coins. The problem simply stipulates that they are different (lighter or heavier) than the true coins. This breaks the solution because you cannot assume that you can just round up to find the weight of a true coin (you might need to round down). Also, some of the hilighted "unique ratios" are ambiguous if you don't know whether the coins are lighter or heavier.

The more complex weighing scheme in the solution [below accounts] for this possibility.

We want to find the the counterfeit pile, the true weight Wt, and the counterfeit weight Wc. Let W1 be the result from weighing the number of coins from each pile indicated in row A of the table below. There are now four cases to be considered:

If W1 is an integer multiple of 35, then the counterfeit coins are from pile 13. Wt = W1/35, and Wc is determined by weighing a single coin from pile 13. Otherwise, W1 will be somewhere between two multiples of 35, M and N. Wt will either be M/35 or N/35. There are now three cases to consider:

Case 1: If  $W1 - M \ge 20$ , then Wt must be N / 35. Let W2 be the result from weighing coins according to table row B. If W2 is an integer multiple of 31, then the counterfeit coins are from pile 5. Four counterfeit coins were included in weighing A, so  $W1 = 31 \times Wt + 4 \times Wc$  and  $Wc = (W1 - 31 \times Wt)/4$ .

Otherwise, let  $D1 = W1 - 35 \times Wt$ . D1 is an integer multiple K1 of (Wc - Wt), K1 = 1, 2, 3, or 4. Let  $D2 = W2 - 31 \times Wt$ . D2 is similarly an integer multiple of (Wc - Wt). The counterfeit coins are from the pile whose ratio of coins in weighing A to coins in weighing B is equal to D1 / D2. If K1 is the number of coins used in weighing A of the counterfeit pile, then  $Wc = (W1 - 31 \times Wt)/K1$ .

Case 2: If W1 - M < 15, then Wt must be M / 35. The results can be calculated as in case 1.

Case 3: If  $15 \ge W1 - M < 20$ , then the difference between W1 and M or N is small enough that either could be the basis for Wt, but big enough that we know that 4 counterfeit coins were included in weighing A. So the counterfeit coins must be in pile 1, 2, 3, 4, or 5. Let W2 be the result from weighing coins according to table row C. If W2 is an integer multiple of 40, then the counterfeit coins are in pile 5. Wt = W2 / 40, and  $Wc = (W1 - 31 \times Wt)/4$ .

Otherwise, W2 will be somewhere between two multiples of 40, P and Q. Since the maximum possible weight difference is less than  $5 \times 4 = 20$ , Wt = (whichever of P or Q is closest to W2) / 40. We can now calculate  $Wc = (W1 - 31 \times Wt)/4$ . The number of counterfeit coins used in weighing C is given by  $K2 = (W2 - 40 \times Wt)/(Wc - Wt)$ , and this will determine whether the counterfeit coins came from pile 1, 2, 3, or 4.

Weighing Table:

Pile:	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
A:	4	4	4	4	4	3	3	3	2	2	1	1	0	35
B:	4	3	2	1	0	4	2	1	4	3	4	3	0	31
C:	4	3	2	1	0	4	4	4	4	4	4	4	2	40