We start with a Bridge Problem from Larry Kells, who wants to know the best chance of making 7 Spades with

S - A
H - AKQ
D - 5
C - J9765432

S - KQJ1098
H -
D - AKQJ432
C -

You are South, the declarer, and the opening lead is a spade, with East following suit. Assume there are no inferences to be had from the bidding or the lead, and that the opponents will make no mistakes for the rest of the play.

Most responders agree that there are two possible lines with nearly the same likelihood of success. The following response is from Jorgen Harmse

Once the lead is in dummy, Declarer must draw trump and play diamonds. The question is whether to cash hearts first. I assume that East's card on Trick 1 is uninformative, for example that the 7 does not suggest a singleton. Cashing one or two hearts does not help to make the contract, and the probability of a favourable diamond split is slightly higher than the probability that 3 hearts can be cashed. Declarer should nevertheless try to cash three hearts: if East ruffs then Declarer can overruff, draw trump, and try diamonds. (If hearts are not ruffed then Declarer ruffs a club, draws trump, and plays the top diamonds.) To compute probabilities, I determine the relevant number of deals of the 26 cards and subtract the number with a 6-0 trump split. For example, the number of 5-0 diamond splits is the number we would calculate before the opening lead minus the number of deals in which both suits split badly with each defender holding one suit or with one defender holding both.

Let N be the number of ways to deal the defenders' cards so each of them has at least one trump. Then

$$N = C(26,13) - 2C(6,0)C(20,13) = 10,245,560$$

where C(p,q) is the number of combinations of p items chosen q at a time, which is

$$\frac{p!}{(p-q)!q!}$$

The probability of a 5-0 diamond split is

$$\frac{2}{N}\left(C(5,0)C(21,13) - C(5,0)C(6,0)C(15,13) - C(5,0)C(6,6)C(15,7)\right) = \frac{393,900}{N} \approx .0384459.$$

Thus, the probability of success by running diamonds is approximately .9615541.

Note that a defender cannot have 8 hearts and 6 trumps. The probability of an 8-2 or worse heart split is therefore

$$\frac{2}{N}\left(\sum_{i=0}^{2} C(10,i)C(16,13-i) - \sum_{i=0}^{2} C(10,i)C(6,6)C(10,7-i)\right) = \frac{403,520}{N} \approx .0393849$$

Thus, the probability that the hearts can be cashed is approximately .9606151. The excellent possibility of recovering if East ruffs makes this the correct play, but puzzle lovers may want more detail.

The number of deals in which East has two hearts is

$$C(10,2)C(16,11)$$

We must subtract the number of those in which trumps or diamonds split badly. The inclusion-exclusion calculation is simplified by two observations: a defender cannot have 2 hearts and 0 trumps, and a defender with 2 hearts and 0 diamonds must have all the outstanding black cards. The probability that Declarer can make despite a third-round heart ruff is therefore

$$\frac{C(10,2)}{N}\left(C(16,11)-C(5,5)C(11,6)-C(6,6)C(10,5)+C(5,5)C(6,6)C(5,0)\right)=\frac{164,475}{N}\approx .0160533.$$

Similar calculations for the 9-1 & 10-0 heart splits show that the probability of making by trying to cash hearts is

$$\frac{10,019,650}{N}\approx .9779504$$