Larry Kells wonders what is the highest high-card point count one can hold, including all four aces, such that there is a distribution of cards among the other three hands for which 3NT is set with best play on both sides,

Guy Steele gave an amazingly thorough analysis including several difficult corner cases. He writes.

The problem does not specify which hand holds the aces, so we consider all four cases. If either defender holds the aces, then that hand may contain the maximum of 37 points (AKQJ AKQ AKQ AKQ) and the 3NT contract will surely be set. (Why didn't that defender overcall 3NT with 7NT? Maybe the game was penny-a-point. Down nine doubled vulnerable is 2600 points; making 7NT wins at most 1770 if opponents don't double, and the aces earn 150 honors either way.)

Now suppose that declarer or dummy holds the aces. Our answer to the problem is 31 points. First we prove that if either declarer or dummy holds 32 points, 3NT can be fulfilled against any defense. Then we exhibit a hand in which either declarer or dummy may hold 31 points and defenders can set 3NT against any strategy.

There are 57 ways one hand can hold 32 high-card points or more while holding all four aces. To save space, we do not list them here, but inspection reveals that such a hand has nine top tricks (and, of course, first-round control in all four suits) in all cases except four. In the three cases AKQJ AKJ AQJ AQJ, AKQ AKQ AQJ AQJ, and AKQ AKJ AKJ AQJ, the hand has eight top tricks and a ninth is easily developed by the strategy of playing the J of any suit headed by AQJ at the first possible opportunity.

The last case to consider is AKJ AKJ AKJ AKJ. If declarer holds these cards, then any opening lead gives declarer a free finesse and declarer easily takes nine tricks. In the unlikely case that dummy holds AKJ AKJ AKJ AKJ, more delicate reasoning is required; in this one case lies most of the complexity of the problem.

If declarer holds any Q, or if E or W has a singleton or doubleton Q, or if W makes his opening lead away from a Q (allowing dummy to win with the J), then dummy can take three tricks in that suit (and nine tricks in all). Also, if E holds Qxx in any suit, then declarer can play A-K-J in that suit (after winning the first trick if it's in another suit); either the J wins, or E is thrown on lead stripped of that suit and must give dummy a free finesse in another suit. Note that if W has all four queens, W must lead away from a Q, and if E has all four queens, then at least one Q is singleton, doubleton, or Qxx.

Now we consider the remaining cases, those where declarer holds no Q, no Q is singleton or doubleton, E and W each hold at least one Q, E has no Qxx, and W does not lead away from a Q.

If E holds three queens, declarer wins the first trick and plays A-K of the suit in which E has no Q; this strips E of that suit and forces E to discard, reducing some suit to Qxx or shorter. Declarer then plays A-K-J of that suit; either the J wins or E is thrown on lead stripped of that suit also, and must give dummy a free finesse.

If E holds two queens and neither is Qxx, then the shortest suit E holds has no more than 2 cards; call this the ''short E suit''. Moreover, because no Q is singleton or doubleton, the shortest suit W holds that has no Q has no more than 3 cards; call this the ''short W suit''. Declarer wins the first trick with an A in dummy, then plays A-K of the short E suit (stripping E of that suit) and A of the other suit in which E has no Q. If in either suit in which E had a Q, E now has no more cards than dummy now has face cards, dummy plays (A)-K-J of that suit; either the J wins or E is thrown on lead stripped of that suit also, and must give dummy a free finesse. Otherwise, dummy plays (A)-K-J of the short W suit (stripping W of that suit); E will be able to take at most 4 tricks in that suit, and then must give dummy a free finesse.

If E holds one Q, that is the suit of the opening lead, and W has no more than four cards in that suit (else W would hold a singleton or doubleton Q). If W has fewer than four cards in that suit, dummy plays the J, which E must win with the Q (else dummy has nine tricks) and then must return that same suit (else give a free finesse), of which dummy plays the A-K; W is now stripped of that suit and has fewer than four cards in some other suit, so dummy plays A-K-J in that suit, winning three tricks or endplaying W. But if W has four cards in the suit of the opening lead, dummy wins the first trick and then plays its little card! If this card is won with a Q, dummy now has nine top tricks; otherwise, if this card is won by E, E must now provide a free finesse; otherwise, if the little card is in a suit other than the opening lead, W now has a doubleton Q and dummy can take A-K-J in that suit and win nine tricks; otherwise, W must return the suit of the opening lead at trick three (else give a free finesse) and dummy plays the J, which E must win with the Q (else dummy has nine tricks) and then must return that same suit (else give a free finesse), which dummy wins and then plays A-K-J in any other suit to endplay W.

This concludes the proof that 3NT can always be fulfilled if dummy holds AKJ AKJ AKJ AKJ. (This is a double-dummy analysis. How declarer can choose the correct line of play over the board is a mystery, but no greater a mystery than why declarer bid notrump before dummy did.)

We have therefore proved if either dummy or declarer holds 32 HCP, 3NT can always be fulfilled. Now consider the following hand, in which declarer holds 31 HCP:

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                 S   -
                 H   654
                 D   98765
                 C   98765
S   832                          S   QJ1097654
H   -                            H   Q10987
D   Q10432                       D   -
C   Q10432                       C   -
                 S   AK
                 H   AKJ32
                 D   AKJ
                 C   AKJ
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This defensive strategy always defeats 3NT: On the first trick, defenders play the S 2 and S 5. If declarer ever leads the S K, W plays the S 3 and E plays the S Q. On any other lead from declarer, a defender wins it if possible, as cheaply as possible; otherwise follows suit if possible, as cheaply as possible; and otherwise discards. W always discards as cheaply as possible, alternating between the minor suits, while E discards a low heart the first time and the fifth time, and otherwise discards the second-cheapest spade (preserving the S 4).

When declarer eventually gives up the lead, there are two cases. If declarer has not yet played the S K, W plays the S 3 and E the S Q, forcing declarer to win the with the S K. When declarer eventually gives up the lead again (defenders now have two tricks), there are two subcases. If declarer has played at most five minor-suit cards, then E has at least three spades left; playing the S J (W leads or follows with the 8) followed by the S 10 and S 4 wins three more tricks. But if declarer has played all six minor-suit cards, then W can win the S 8 (E playing the S 4) and the D 10 and C 10 are both good.

If the S K has already been played when declarer first gives up the lead, there are two subcases. If E has four or more spades, they are good; E overtakes the S 8, then takes three more. If E has fewer than four spades, then E made at least four discards, and there are two subsubcases. If declarer played the D A, D K, C A, and C K before giving up the lead, then W can win five tricks with the S 8 (E playing the S 4), D Q, D 10, C Q, and C 10 (if declarer exited with a heart, he goes down two). But if declarer played only three of the top minor-suit honors, the fourth discard by E must have been on the D J or C J. Because declarer did not give up the lead by playing a heart, declarer has played at most two hearts, therefore W has discarded at most twice, once from each minor suit, and so holds both the Q and 10 in the suit of the J declarer led. W wins the J with the Q, plays the 10 (E discarding the H Q), and then E overtakes the S 8 to win three spade tricks.

This strategy also works if declarer’s and dummy’s hands are switched and/or defenders’ hands are switched.

Appendix: the 57 ways to hold four aces and 32 or more HCP total

37 AKQJ AKQ AKQ AKQ
36 AKQJ AKQJ AKQ AK

36 AKQJ AKQ AKQ AKJ
36 AKQ AKQ AKQ AKQ
35 AKQJ AKQJ AKQ AQ
35 AKQJ AKQ AKQ AQJ
35 AKQJ AKQJ AKJ AK
35 AKQJ AKQ AKJ AKJ
35 AKQJ AKQ AKQ AK
35 AKQ AKQ AKQ AKJ
34 AKQJ AKQJ AKQJ A
34 AKQJ AKQJ AKQ AJ
34 AKQJ AKQJ AKJ AQ
34 AKQJ AKQJ AK AQJ
34 AKQJ AKQ AKJ AQJ
34 AKQJ AKQ AKQ AQ
34 AKQ AKQ AKQ AQJ
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34 AKQ AKQ AKQ AK
33 AKQJ AKQJ AQJ AQ
33 AKQJ AKQ AQJ AQJ
33 AKQJ AKQJ AKJ AJ
33 AKQJ AKJ AKJ AQJ
33 AKQJ AKQJ AKQ A
33 AKQJ AKQ AKQ AJ
33 AKQJ AKQJ AK AQ
33 AKQJ AKQ AKJ AQ
33 AKQJ AKQ AK AQJ
33 AKQ AKQ AKJ AQJ
33 AKQ AKQ AKQ AQ
33 AKQJ AKJ AKJ AK
33 AKQ AKJ AKJ AKJ
33 AKQJ AKQ AK AK
33 AKQ AKQ AKJ AK
32 AKQJ AKQJ AQJ AJ
32 AKQJ AKJ AQJ AQJ
32 AKQJ AKQJ AQ AQ
32 AKQJ AKQ AQJ AQ
32 AKQ AKQ AQJ AQJ
32 AKQJ AKQJ AKJ A
32 AKQJ AKQJ AK AJ
32 AKQJ AKQ AKJ AJ
32 AKQJ AKJ AKJ AQ
32 AKQJ AKJ AK AQJ
32 AKQ AKJ AKJ AQJ
32 AKQJ AKQ AKQ A
32 AKQ AKQ AKQ AJ
32 AKQJ AKQ AK AQ
32 AKQ AKQ AKJ AQ
32 AKQ AKQ AK AQJ
32 AKJ AKJ AKJ AKJ
32 AKQJ AKJ AK AK

32 AKQ AKJ AKJ AK
32 AKQ AKQ AK AK