Mr. Allan Gottlieb
New York University
715 Broadway, 10th Floor
New York, NY 10003

5/29/02

Dear sir,

This letter is in reply to statements concerning the need for mathematical proof of a solution to a puzzle in the Dec. 2001 puzzle corner. The puzzle labeled 'Dec. 2' asked: Can more squares of area 1 or equilateral triangles of area 1 be fit in a circle of radius R>>1? I replied and stated that more squares could be placed within the circle. Below is my mathematical reasoning for this conclusion.

First we consider the equilateral triangles dimensions. The base length of the triangle is designated as B and it's height H. The triangle's area BxH/2=1. All three angles of an equilateral triangle are 60 degrees so that 2xH/B=tan 60deg. Solving these two equations we find: $H=\sqrt{\tan 60} = 1.316074013$ and $B= 2/H = 1.519671371$.

We start here explaining how the number of triangles is determined. A series of horizontal lines are spaced H apart vertically to divide the circle into areas where the triangles are placed. The lowest line is set at -R+KxH for the triangles and -R+K for the squares where K is a number between 0 and 1. Two horizontal areas are shown in the accompanying chart. The length of the lines defining the areas are named S for the shorter line and L for the longer line. The length of any of these lines is determined as the square root of the square of the circle's radius minus the square of the height of the line above or below the horizontal line passing through the circle's center.

The equations used to determine the number of triangles with their bases along the lines marked S and L are printed on the accompanying chart.. The upper set of triangles applies when L-S>B and a line tangent to the circle has an elevation angle less than 60 degrees. The lower set of triangles applies when L-S<B and the tangent elevation angle is greater than 60 degrees. NS and NL refer to the number of triangles along the shorter and longer lines. The strange symbol, $\llcorner\!\lrcorner$,in front of the right hand side of an equation means that its fractional part is deleted. The sum of NS and NL gives the total number of triangles that can be placed between the two adjacent horizontal lines. The strange equation for NS on the lower set of triangles is used so that the inverted triangles will perfectly match the areas between the upright triangle set along the lower line of that set. Depending on the circle's radius and the height of the set of two adjacent lines the lower set of triangles can have the same as or one more triangle than the number of inverted triangles along it's upper line. The upper set of triangles however always has exactly one more triangle along it's lower line than along its upper line.

Explanation of the number of squares between two adjacent horizontal lines is much simpler. Since each square has a width of 1 unit, the number of squares, NS, is simply equal to S/B with it's fractional part removed. Again the number of squares between each set of adjacent horizontal lines is summed to get the total number of squares inside the circle.

I hope the above has mathematically explained how I arrived at my answer. I have enclosed several papers listing the results for sets of input parameters. On these sheets

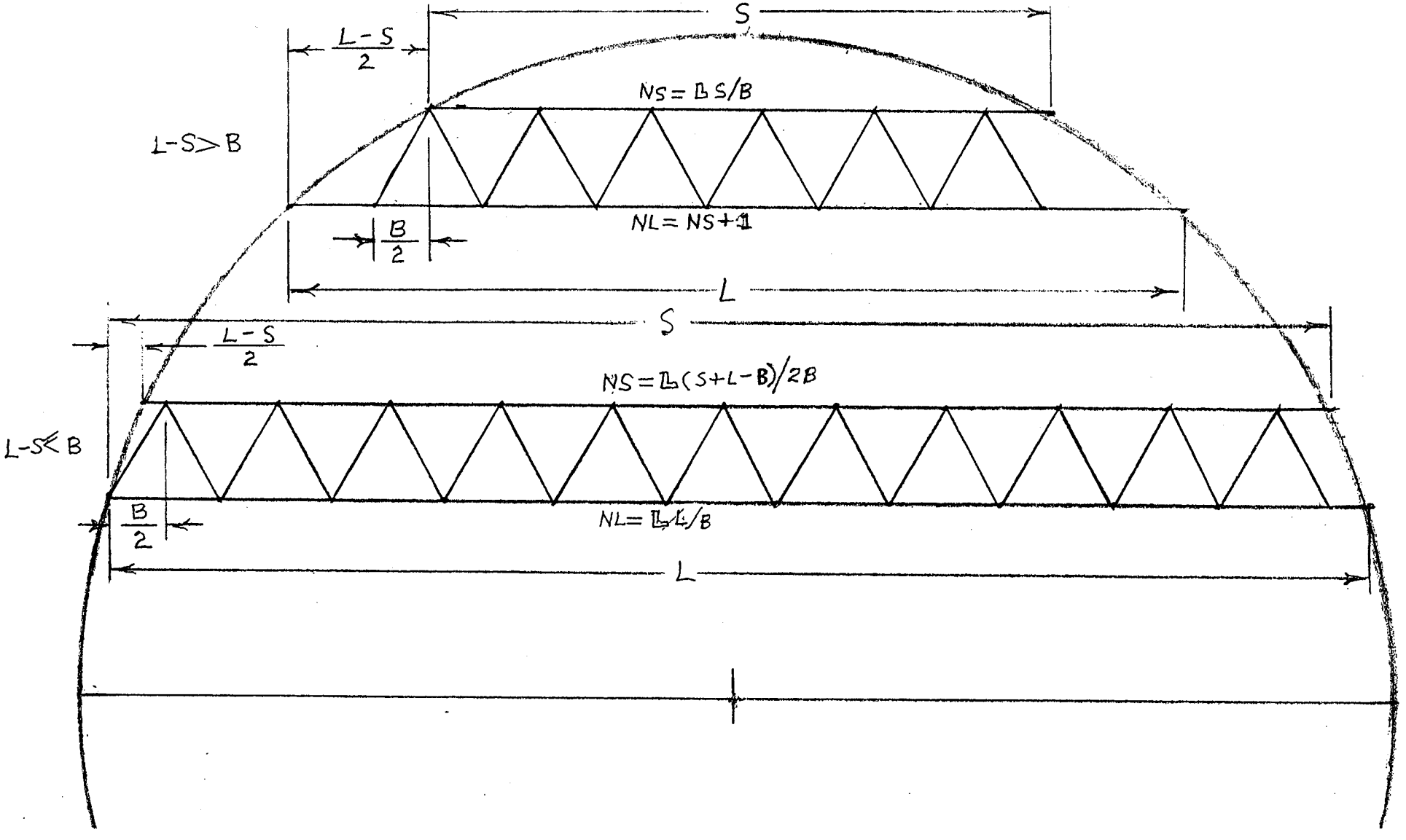
"Number" refers to the number of triangles or squares within the circle. "Percent filled" is the Number divided by the area of the circle. "Area empty" gives the space in square units left empty after putting in the triangles or squares. It is easy to see that although they are very close in the numbers that the squares exceed the triangles. If you would like more information concerning my methods please let me know by mail or phone. I will gladly send you a copy of the computer program used to generate these numbers.

sincerely,
Charles E Muehe

Charles E. Muehe
13 Overlook Drive
Atkinson, NH 03811

Phone: 603 362 5695

P.S. My address and telephone number have changed since the last letter I sent to you.


S
(L-S)/2
NS = ⌊S/B
L-S ≥ B
NL = NS+1
B/2
L
S
(L-S)/2
NS = ⌊(S+L-B)/2B
L-S < B
B/2
NL = ⌊L/B
L

WS PUZZDEC2 FUNCTION table 5/29/02 11:12AM

Radius of circle	K	TRIANGLES Number	Percent filled	Area empty	SQUARES Number	Percent filled	Area empty
10.0	0.5	284.0	90.4	30.2	285.0	90.7	29.2
20.0	0.5	1189.0	94.6	67.6	1199.0	95.4	57.6
30.0	0.5	2730.0	96.6	97.4	2735.0	96.7	92.4
40.0	0.5	4899.0	97.5	127.5	4901.0	97.5	125.5
50.0	0.5	7704.0	98.1	150.0	7709.0	98.2	145.0
60.0	0.5	11121.0	98.3	188.7	11131.0	98.4	178.7
70.0	0.5	15163.0	98.5	230.8	15175.0	98.6	218.8
80.0	0.5	19851.0	98.7	255.2	19863.0	98.8	243.2
90.0	0.5	25159.0	98.9	287.9	25173.0	98.9	273.9
100.0	0.5	31103.0	99.0	312.9	31111.0	99.0	304.9

WS PUZZDEC2 FUNCTION table 5/29/02 10:22AM

Radius of circle	K	TRIANGLES Number	TRIANGLES Percent filled	TRIANGLES Area empty	SQUARES Number	SQUARES Percent filled	SQUARES Area empty
48.2	0.1	7142.0	97.9	156.7	7159.0	98.1	139.7
48.4	0.1	7203.0	97.9	156.4	7216.0	98.1	143.4
48.6	0.1	7259.0	97.8	161.3	7283.0	98.1	137.3
48.8	0.1	7328.0	97.9	153.5	7336.0	98.1	145.5
49.0	0.1	7386.0	97.9	157.0	7393.0	98.0	150.0
49.2	0.1	7448.0	97.9	156.7	7459.0	98.1	145.7
49.4	0.1	7506.0	97.9	160.6	7518.0	98.1	148.6
49.6	0.1	7564.0	97.9	164.8	7583.0	98.1	145.8
49.8	0.1	7629.0	97.9	162.3	7647.0	98.1	144.3
50.0	0.1	7687.0	97.9	167.0	7697.0	98.0	157.0
50.2	0.1	7760.0	98.0	156.9	7772.0	98.2	144.9
50.4	0.1	7816.0	97.9	164.1	7826.0	98.1	154.1
50.6	0.1	7880.0	98.0	163.6	7889.0	98.1	154.6
50.8	0.1	7942.0	98.0	165.3	7957.0	98.1	150.3
51.0	0.1	8013.0	98.1	158.3	8016.0	98.1	155.3
51.2	0.1	8065.0	97.9	170.5	8090.0	98.2	145.5
51.4	0.1	8133.0	98.0	167.0	8147.0	98.2	153.0
51.6	0.1	8197.0	98.0	167.7	8211.0	98.2	153.7
51.8	0.1	8267.0	98.1	162.6	8276.0	98.2	153.6
52.0	0.1	8326.0	98.0	168.9	8341.0	98.2	153.9

WS puzzdec2 FUNCTION table 5/29/02

Radius of circle	K	TRIANGLES Number	TRIANGLES Percent filled	TRIANGLES Area empty	SQUARES Number	SQUARES Percent filled	SQUARES Area empty
10.0	0.0	281.0	89.4	33.2	284.0	90.4	30.2
20.0	0.0	1193.0	94.9	63.6	1194.0	95.0	62.6
30.0	0.0	2724.0	96.3	103.4	2730.0	96.6	97.4
40.0	0.0	4895.0	97.4	131.5	4900.0	97.5	126.5
50.0	0.0	7671.0	97.7	183.0	7700.0	98.0	154.0
60.0	0.0	11112.0	98.3	197.7	11122.0	98.3	187.7
70.0	0.0	15167.0	98.5	226.8	15170.0	98.5	223.8
80.0	0.0	19848.0	98.7	258.2	19848.0	98.7	258.2
90.0	0.0	25149.0	98.8	297.9	25170.0	98.9	276.9
100.0	0.0	31065.0	98.9	350.9	31104.0	99.0	311.9

WS PUZZDEC2 FUNCTION table 5/29/01 10:42PM

Radius of circle	K	TRIANGLES Number	Percent filled	Area empty	SQUARES Number	Percent filled	Area empty
96.0	0.5	28652.0	99.0	300.9	28671.0	99.0	281.9
97.0	0.5	29242.0	98.9	317.2	29277.0	99.0	282.2
98.0	0.5	29846.0	98.9	325.9	29875.0	99.0	296.9
99.0	0.5	30464.0	98.9	326.7	30493.0	99.0	297.7
100.0	0.5	31103.0	99.0	312.9	31111.0	99.0	304.9
101.0	0.5	31717.0	99.0	330.4	31745.0	99.1	302.4
102.0	0.5	32352.0	99.0	333.1	32377.0	99.1	308.1
103.0	0.5	32994.0	99.0	335.2	33021.0	99.1	308.2
104.0	0.5	33654.0	99.0	325.5	33669.0	99.1	310.5
105.0	0.5	34300.0	99.0	336.1	34323.0	99.1	313.1