Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

Back to Brooklyn. In the early 1950s I was an avid fan of the Brooklyn Dodgers even though most of my family favored the Yankees. Although the Dodgers are not destined to return to Brooklyn, it is likely that I will replace them there.

Alice and I have given up our small Manhattan apartment and are very likely to soon complete the purchase of a Brooklyn town house, which we shall share with our younger son, Michael, his lovely wife, Maureen, and their dog, Cersei.

Problems

M/J1. We start with a "football" problem from Steven Minsker. (Think soccer with four quarters.) Suppose a game in which team A scores p_1, p_2, p_3, p_4 points in the first, second, third, and fourth quarters respectively, while team B scores q_1, q_2, q_3, q_4 points. At the end of regulation time, the game is tied. In sudden-death overtime, team A scores p^* points and wins the game. If these nine quantities are all distinct primes, what is the lowest possible final score, and what values for the nine quantities achieve this?

M/J2. Arthur Wasserman imagines taking a positive integer and moving its last digit to the front. For example 1,234 would become 4,123. Wasserman wants to know: What is the smallest positive integer such that when you do this, the result is exactly double the original?

M/J3. Steve Peters isn't too fussy about wearing paired socks. He writes: Imagine you have 16 socks in 15 colors. After doing laundry, you randomly pick two clean socks to wear each day until you run out and need to do the laundry again. How many days, on average, do you wear unmatched socks between days when you inadvertently wear the matching pair?

Speed department

Sorab Vatcha wants you to find two five-digit even numbers that have the same digits but in reverse order. There are no repeated digits or zeros, and the larger number is four times the smaller.

Solutions

J/F1. Andy Schwartz must not have taken the math sequence in the same order I did. To help his son calculate the integral of $\sqrt{1 + x^2}$, he left the beaten path and tried the substitution $x = i\sin(u)$. His challenge for us is to find a complex analytic expression for $\arcsin(x)$.

The following interpretation and solution is from Tony Trojanowski.

If x is a real number such that $-1 \le x \le 1$, there are infinitely many real numbers y such that $\sin y = x$; however, there is only one such y satisfying $-\pi/2 \le y \le \pi/2$. This unique value of y is the *inverse sine* of x: $y = \sin^{-1} x$. Since $\sin y = x$, $\cos^2 y = 1 - x^2$, and because the cosine is non-negative for angles y in the range $-\pi/2 \le y \le \pi/2$, we may write $\cos y = \sqrt{1 - x^2}$ unambiguously. By Euler's identity

$$\begin{aligned} e^{iy} &= \cos y + i \sin y \\ &= \sqrt{1 - x^2} + ix \end{aligned}$$

Notice that for $-\pi/2 \le y \le \pi/2$, e^{iy} is a complex number of modulus 1 in the closed right half-plane. If z is a non-zero complex number in the closed right half-plane, z can be written

$$z = re^{i\theta}$$

where $r > 0$ and $-\pi/2 \le \theta \le \pi/2$. For such z we can define

$$\ln z = \ln r + i\theta$$

Then

$$\ln e^{iy} = iy = \ln(\sqrt{1 - x^2} + ix)$$

and

$$-\pi/2 \le y \le \pi/2.$$

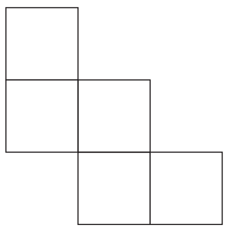
So finally

$$\sin^{-1} x = y = -i \ln(\sqrt{1 - x^2} + ix)$$

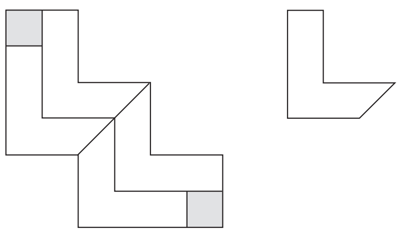
as desired.

J/F2. Our last regular problem is another "modest polyomino" from Richard Hess and Robert Wainwright.

You are to design a connected tile so that three of them cover the shape below. The tiles are identical in size and shape and may be turned over so that some are mirror images of the others. They must not overlap each other or the border of the polyomino.



We have published several polyomino problems in the past; this time we received no solutions except from the problem authors. I believe this is because the solution tile is not rectilinear.



The left diagram shows the division into tiles, and the right diagram illustrates an individual tile with its diagonal edge.

Better late than never

Y2021. John Chandler significantly improved the published solution. He writes:

The following are minimal, unlike those published.	The following have digits in the preferred order, unlike those published.	The following were missing from the published list.
$1 = 1^{202}$	$10 = 20/2 \times 1$	$12 = 22 - 10$
$2 = 2 + 0^{21}$	$19 = 20 - 2 + 1$	$16 = 2 \times (10 - 2)$
$3 = 21^{0} + 2$	$22 = 20 + 2^{1}$	$22 = 20 + 2^{1}$
$4 = (10 - 2)/2$	$23 = 20 + 2 + 1$	$25 = (10/2)^{2}$
$8 = 20 - 12$	$39 = 20 \times 2 - 1$	$38 = 2 \times (20 - 1)$
	$42 = (2 + 0) \times 21$	$64 = (2 - 10)^{2}$

Other responders

T. Bielecki, W. Blank, M. Brill, S. Brown, J. Chandler, M. Coiley, T. Gauss, H. Grossk, J. Larsen, J. Licini, T. Mita, T. Maloney, J. Milgram, R. Morgen, J. Rulnick, W. Sargent, P. Scott, M. Thomas, B. Wake, A. Yen, and Zachary.

Solution to speed problem

21,978 and 87,912