Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

Introduction

My world continues its return to normalcy. Two years ago I was on sabbatical, and last year NYU was all remote via Zoom. As a result, yesterday was the first time in over two years that I was in the same room as my students. During my absence, the system in the main lecture hall was changed, and I could not determine how to show my electronic lecture notes on the screen. The department has a help desk populated with computer science majors who have always been able to show me what I'm doing wrong, but yesterday, in the wake of Hurricane Ida, they could not get to the university—so I was on my own. My appreciation for the help desk, already high, has increased significantly.

Since I have not done so for a while, let me review the ground rules for this column. In each issue I present three regular problems, the first of which is normally related to bridge, chess, or some other game, and one "speed" problem. Readers are invited to submit solutions to the regular problems, and two issues later, one solution is printed for each. Please try to send your solutions early. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other responders" section. Major corrections or additions to published solutions are sometimes printed in the "Better late than never" section.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same issue in which the problem is published. There is also an annual problem, published in the first issue of each calendar year, and sometimes I go back into history to republish problems that have remained unsolved.

My longtime editor, Alice Dragoon, is taking a medical leave this issue. I join the TR staff in wishing her a full and speedy recovery. This issue of Puzzle Corner is dedicated to her.

Problems

N/D1 Daniel Glickman wants us to analyze a variant of the basketball shooting game H-O-R-S-E between Daniel and his friend Randy. Daniel and Randy have equal shooting skills. That means for any given basketball shot, they have the same probability of success. The rules are:

Randy shoots first, and then they alternate.

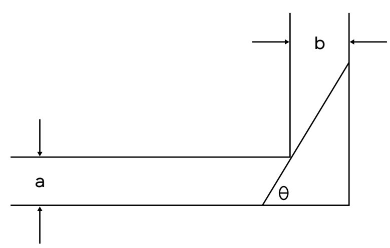
1. Initially Randy chooses the shot they must make.
2. If Randy makes the shot and Daniel misses the (identical) shot, Randy wins.
3. If Randy makes the shot and Daniel makes the shot, Daniel wins.
4. If Randy misses the shot, then Daniel has control; he chooses the next shot and assumes the position Randy had at the beginning.

The question is: How difficult a shot should Randy choose initially? Clearly choosing a 0% shot just gives control to Daniel. Also, choosing a trivial shot leads to a loss for Randy.

N/D2 Our second problem is from Pericles Manglis, who wants you to find the *n*th power and the *n*th root of the simple square matrix

$$\begin{matrix} (A + B)/2 & (A - B)/2 \\ (A - B)/2 & (A + B)/2 \end{matrix}$$

N/D3 I misplaced the cover letter that came with the following problem (and others). I will acknowledge authorship if the proposer contacts me (and forgives my oversight).



What is the longest ladder that can be taken around a two-dimensional 90° corner between two halls of width a and b?

Speed department

SD Sid Shapiro flips a fair coin repeatedly, stopping when one of these sequences occurs: HHHH or THHH. Which sequence is more likely to occur?

Solutions

J/A1 George Fisher likes the card game euchre, where the deck comprises the highest six cards of each suit. In late 2019, for the first time in his life, George was dealt a five-card hand with all the cards in the same suit. He asks how the probability of being dealt five cards of only one suit with the euchre deck compares with the probability using a standard 52-card deck. He also wonders if there is a number of suits with the same expectation (say, within 1%) with both the euchre and the standard card deck. He also considers a minimalist deck with only two cards in each suit and wonders: With this deck, what is the probability of getting a hand with exactly one, two, three, or four suits?

John Chandler sent us the following solution. With a euchre deck the probability of getting all five cards in the same suit is $(5/23)(4/22)(3/21)(2/20) = 5.6 \times 10^{-4}$. With a normal 52-card deck it's $(12/51)(11/50)(10/49)(9/48) = 2 \times 10^{-3}$. With a minimalist deck (the game would have to be solitaire, by the way), it's impossible to get all five cards in one suit, or even in two suits. The probability of getting all four suits is the same as the probability of dealing out three cards all in different suits, i.e., $(6/7)(4/6) = .571$, and thus the probability of getting exactly three suits in a five-card hand is .429. As for the comparison between the euchre and 52-card decks: yes, there is nearly the same probability of getting three suits in both cases, .584 for the former and .588 for the latter. (And of course, the probability of getting only one suit was already shown to be nearly zero for both decks.)

J/A2 This was another of Richard Hess's "logical hat problems." In each of these problems, logicians are each wearing a hat with a number. The logicians see the number on every other hat, but not the one on their own. Each logician is error-free in his or her reasoning and is given that information before the puzzle starts.

For this particular puzzle, there are three logicians, A, B, and C. All three know that each number is a positive integer and that one of the numbers is the sum of the other two.

A announces, "I don't know my number," after which B announces, "My number is 10." What numbers are on A and C?

Norman Derby polished this one off fairly quickly.

Since the three numbers are positive integers, the sum of two of them is larger than either one. So if A looks around and notices that B and C have different numbers, he will realize that there are two possibilities: 1) his number is the largest (equal to the sum of B and C) or 2) the larger of B or C is the largest, and his number is the difference between them. There is no way for him to decide between these possibilities, so he would announce that he does not know what his number is. On the other hand, if B notices that A and C each have the same number, then he will know that his must be the largest and hence equal to the sum of A and C. Since B says that he knows his number is 10, then A and C are both 5.

J/A3 William Stein offered us the following sobering puzzle.

The probability of a US antimissile missile hitting an incoming intercontinental ballistic missile (ICBM) is relatively low, typically between 0.5 and 0.6. We call this probability a. Therefore, it is US doctrine to fire a "high number" m of antimissile missiles at each ICBM that we want to stop from landing in our territory. What is the probability of an ICBM getting through given a and m?

If we are attacked by N incoming ICBMs, what is the probability of at least one getting through given a, N, and m? (This requires $m \times N$ antimissile missiles.)

Peter Kramer first states his assumption that the answer to this problem is not classified under the Atomic Energy Act of 1946. He then writes:

If a is the probability of one antimissile missile hitting a targeted incoming ICBM, then $(1 - a)$ is the probability of that antimissile missile missing. $(1 - a)m$ is the probability of m antimissile missiles all missing the incoming ICBM and the ICBM getting through.

$1 - (1 - a)m$ is the probability of stopping that one incoming ICBM with at least one antimissile missile. (Actually, it will never be more than one, since the problem is different after the incoming ICBM is hit.) If there are N incoming ICBMs, then the probability of stopping them all is $(1\ (1\ a)m)N$ and the probability of not stopping them all (i.e., at least one incoming ICBM getting through) is $1\ (1\ (1\ a)m)N$.

The expected number that will get through is $N \times (1 - a)m$.

If (these numbers are just speculative guesses) $a = 0.5$ and $m = 10$ and $N = 1{,}000$, then using 10,000 antimissile missiles against 1,000 incoming ICBMS:

- the probability of stopping any one incoming ICBM is 0.9990234
- the probability of stopping them all is 38%
- the probability of not stopping them all is 62%
- the expected number getting through is 0.98 (a.k.a. 1).

Other responders

S. Berkenblit, M. Brill, M. Chartier, T. Chow, F. Cogswell, G. Coram, G. Fischer, J. Freilich, M. Gordy, T. Hafer, J. Hardis, P. Hartmann, S. Helm, A. Hirshberg, H. Hodara, B. Krauss, J. Larsen, T. Malony, D. Mellinger, T. Mita, G. Muldowney, B. Rhodes, K. Rosato, R. Rosen, J. Rulnick, E. Signorelli, S. Silberberg, S. Sperry, W. Stein, A. Stern, U. Sukhatme, T. Woods, and J. Wrinn

Proposer's solution

THHH wins 15/16 of the time, since HHHH can only win on the fourth toss.