Getting back to normal—at least in the northeast US.

Last issue I reported our first restaurant visit in a long time. This time we celebrate visiting an out-of-state friend. The celebration indicates how cloistered we have all been for over a year. It is about time! I hope everyone will be back to normal soon.

Problems

S/O1 Apparently some chess players consider the ordinary rules dull and offer variations for us to try. One variant, sometimes called “free-capture chess,” permits players to capture their own pieces and pawns (as well as the opponent’s). However, players cannot check or capture their own king. Consider the following position in free-capture chess, with White to move.

How does White force a mate in two?

S/O2 Richard Lipes sent us the following number problem. Alice lives on a long street in which all the houses are on one side, and the houses are numbered consecutively (i.e., 1, 2, ... N). Alice, a numbers whiz, noticed that the sum of all the house numbers below her house equals the sum of all the house numbers above her house. Her house number has three digits. What is it?

S/O3 The following two-part cryptarithmetic problem is from David Singmaster’s book *Problems for Metagrobologists*. Can two ODDS make an EVEN in the cryptarithmic sense? That is, try to substitute digits for letters and solve

\[ \text{ODD} + \text{ODD} = \text{EVEN} \]

Part two is to arrange for three ODDs to make an ODD cryptarithmetically.

Speed department

Ermanno Signorelli wants you to find an arithmetic expression evaluating to 20 that contains three 9s, no other digits, and any standard arithmetic operators that you wish.

Solutions

M/J1 Larry Kells wants you to find a legal chess game where White has all 16 starting pieces, Black has only his king, it is White’s move, and White has been stalemated.

Team Just ∫ from Newark Academy in Livingston, New Jersey, sent us the following solution.

We started by finding the end position in which White was stalemated. We chose the position below because it had the smallest number of doubled pawns, resulting in a solution with the fewest moves. After creating the end position, we then attempted to solve the position in as few moves as possible. Our move list is given in standard chess notation.
Move list:

1. d4 c5  
2. dxc5 d5  
3. e4 e5  
4. exd5 Bd6  
5. f4 f5  
6. fxe5 Be6  
7. g4 g5  
8. gxf5 Nf6  
9. fxe6 g4  
10. h3 O-O  
11. hxg4 h6  
12. Bxh6 Ne4  
13. a4 Nc6  
14. Bg7 Nc3  
15. Ra3 Nd4  
16. Rh7 Ncb5  
17. Rf3 Rf7  
18. Rxf7 Bf8  
19. Bxf8 Qe7  
20. Rxe7 Re8  
21. Rxe8 Nc7  
22. Rd7 a6  
23. Bxa6 b6  
24. Bc8 b5  
25. Rdd8 Nf3+  
26. Nxh3 b4  
27. Nd4 b3  
28. c4 Nb5  
29. Nc6 Nc7  
30. Nb8 Nb5  
31. Qxb3 Nc7  
32. Qb7 Nb5  
33. Nc3 Nc7  
34. Nb5 Kh8  
35. Nxc7 Kg8  
36. Na8 Kh8  
37. Kd2 Kg8  
38. Rf3 Kg7  
39. Kb4 Kg8  
40. Ka5 Kh8  
41. Ka6 Kg8  
42. Ka7 Kh8  
43. a5 Kg8  
44. a6 Kh8  
45. b4 Kg8  
46. b5 Kh8  
47. b6 Kg8  
48. c6 Kh8  
49. c7 Kg8  
50. c5 Kh8  
51. c6 Kg8  
52. d6 Kh8  
53. d7 Kg8  
54. e7 Kh8  
55. e6 Kg8  
56. g5 Kh7  
57. g6+ Kh8  
58. g7+ Kg8  

A right triangle is identified having its longer leg coincident with one side of the hexagon plus the sides of two projections, which total three length units. In the same right triangle, the hypotenuse and shorter leg are coincident with the containing equilateral triangle and thus subtend an angle of $\pi/3$. The hypotenuse is therefore $(2/\sqrt{3})$ times the longer leg, or $2\sqrt{3}$ units; the shorter leg is half this length, or $\sqrt{3}$. The side of the containing triangle comprises one long and one short leg, which sum to $3\sqrt{3}$ units. The areas of the triangle and hexagon are then found using the formula for an equilateral triangle:

$$A_T = \frac{(3\sqrt{3})^2 \sqrt{3}}{4} = \frac{27\sqrt{3}}{4}$$

$$A_H = 6 \cdot \frac{(1)^2 \sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$$

Therefore:

$$\frac{A_H}{A_T} = \frac{(3\sqrt{3})/2}{27\sqrt{3}/4} = \frac{2}{9}$$

Thus the hexagon area is $2/9$ that of the containing triangle. The full hexagram area is twice this, or $4/9$.

### Better late than never

**J/F 2** Loren Bonderson notes that the first term in the final formula for $L$ is missing a square root.

### Other responders


### Solution to speed problem

$$\frac{\sqrt{9} + \sqrt{9}}{\sqrt{9}} = \frac{9}{3} = 3$$

M/J2 Tony Yen overheard the following problem in a Chinese airport lounge.

You are given that $AD = DB$, $FC = 3AF$, and $EC = 2BE$. Find the area of triangle $ABC$ in terms of the area of triangle $DEF$.

John Chandler knows that the area of any triangle equals $\frac{1}{2}$ the sine of any of its angles times the product of the lengths of the two adjacent sides. He then notes that in terms of area, $ADF = ABC/8$, $BDE = ABC/6$, and $FEC = ABC/2$. Thus, $DEF = ABC(1 - 1/8 - 1/6 - 1/2) = ABC/5(24)$, or $ABC = 24/5$ DEF.

For those of you (like me) who didn’t know the area formula Chandler used, I posted Ed Koch’s solution on the Puzzle Corner website (/cs.nyu.edu/gottlieb/tr).

M/J3 Sorab Vatcha has an equilateral triangle containing the largest regular hexagram that can fit inside. What is the ratio of the area of the hexagon to the area of the triangle?

Greg Muldowney sent us a clear solution accompanied by a clear diagram.

I happily include both below.

The largest hexagram that fits in an equilateral triangle is one in which all six projections from the sides of the central hexagon have their apices located on the triangle perimeter. If the side of the hexagon is assigned unit length, then the sides of each triangular projection are also of unit length.

---

**Solution to speed problem**

$$\frac{\sqrt{9} + \sqrt{9}}{\sqrt{9}} = \frac{9}{3} = 3$$