Puzzle corner

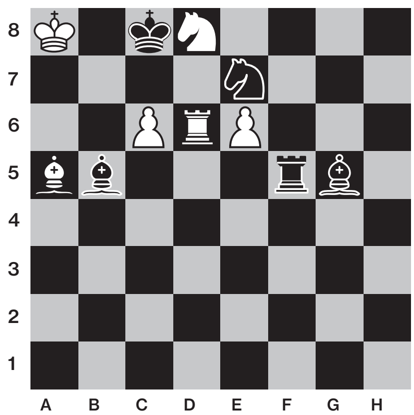
Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

Getting back to normal—at least in the northeast US.

Last issue I reported our first restaurant visit in a *long* time. This time we celebrate visiting an out-of-state friend. The celebration indicates how cloistered we have all been for over a year. It is about time! I hope everyone will be back to normal soon.

Problems

S/O1 Apparently some chess players consider the ordinary rules dull and offer variations for us to try. One variant, sometimes called "free-capture chess," permits players to capture their own pieces and pawns (as well as the opponent's). However, players cannot check or capture their own king. Consider the following position in free-capture chess, with White to move.



How does White force a mate in two?

S/O2 Richard Lipes sent us the following number problem. Alice lives on a long street in which all the houses are on one side, and the houses are numbered consecutively (i.e., 1, 2, ... N). Alice, a numbers whiz, noticed that the sum of all the house numbers below her house equals the sum of all the house numbers above her house. Her house number has three digits. What is it?

S/O3 The following two-part cryptarithmetic problem is from David Singmaster's book *Problems for Metagrobologists*. Can two ODDs make an EVEN in the cryptarithmetic sense? That is, try to substitute digits for letters and solve

`ODD + ODD = EVEN`

Part two is to arrange for three ODDs to make an ODD cryptarithmetically.

Speed department

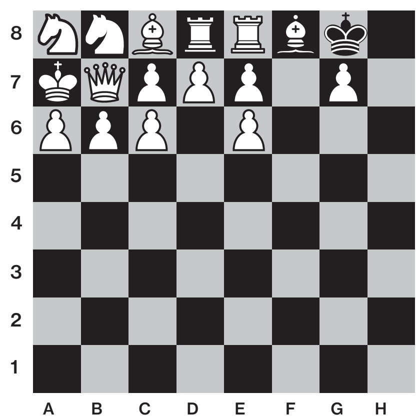
Ermanno Signorelli wants you to find an arithmetic expression evaluating to 20 that contains three 9s, no other digits, and any standard arithmetic operators that you wish.

Solutions

M/J1 Larry Kells wants you to find a legal chess game where White has all 16 starting pieces, Black has only his king, it is White's move, and White has been stalemated.

Team Just ∫*du* It from Newark Academy in Livingston, New Jersey, sent us the following solution.

We started by finding the end position in which White was stalemated. We chose the position below because it had the smallest number of doubled pawns, resulting in a solution with the fewest moves. After creating the end position, we then attempted to solve the position in as few moves as possible. Our move list is given in standard chess notation.

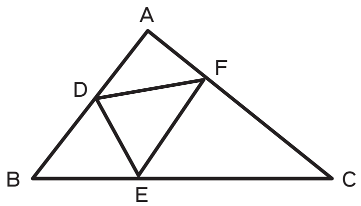


Move list:

1. d4 c5
2. dxc5 d5
3. e4 e5
4. exd5 Bd6
5. f4 f5
6. fxe5 Be6
7. g4 g5
8. gxf5 Nf6
9. fxe6 g4
10. h3 O-O
11. hxg4 h6
12. Bxh6 Ne4
13. a4 Nc6
14. Bg7 Nc3
15. Ra3 Nd4
16. Rh7 Ncb5
17. Rf3 Rf7
18. Rxf7 Bf8
19. Bxf8 Qe7
20. Rxe7 Re8
21. Rxe8 Nc7
22. Rd7 a6
23. Bxa6 b6
24. Bc8 b5
25. Rdd8 Nf3+
26. Nxf3 b4
27. Nd4 b3
28. c4 Nb5
29. Nc6 Nc7
30. Nb8 Nb5
31. Qxb3 Nc7
32. Qb7 Nb5
33. Nc3 Nc7
34. Nb5 Kh8
35. Nxc7 Kg8
36. Na8 Kh8
37. Kd2 Kg8
38. Kc3 Kh8
39. Kb4 Kg8
40. Ka5 Kh8
41. Ka6 Kg8
42. Ka7 Kh8
43. a5 Kg8
44. a6 Kh8
45. b4 Kg8
46. b5 Kh8
47. b6 Kg8
48. c6 Kh8
49. c7 Kg8
50. c5 Kh8
51. c6 Kg8
52. d6 Kh8
53. d7 Kg8
54. e7 Kh8
55. e6 Kg8
56. g5 Kh7
57. g6+ Kh8
58. g7+ Kg8

M/J2 Tony Yen overheard the following problem in a Chinese airport lounge.

You are given that AD=DB, FC=3AF, and EC=2BE. Find the area of triangle ABC in terms of the area of triangle DEF.



John Chandler knows that the area of any triangle equals ½ the sine of any of its angles times the product of the lengths of the two adjacent sides. He then notes that in terms of area, ADF = ABC/8, BDE = ABC/6, and FEC = ABC/2. Thus, DEF = ABC(1 – 1/8 – 1/6 – 1/2) = ABC(5/24), or ABC = 24/5 DEF.

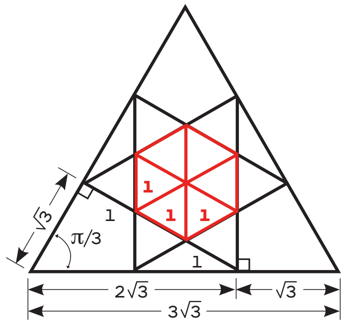
For those of you (like me) who didn't know the area formula Chandler used, I posted Ed Koch's solution on the Puzzle Corner website (//cs.nyu.edu/gottlieb/tr).

M/J3 Sorab Vatcha has an equilateral triangle containing the largest regular hexagram that can fit inside. What is the ratio of the area of the hexagon to the area of the triangle?

Greg Muldowney sent us a clear solution accompanied by a clear diagram.

I happily include both below.

The largest hexagram that fits in an equilateral triangle is one in which all six projections from the sides of the central hexagon have their apices located on the triangle perimeter. If the side of the hexagon is assigned unit length, then the sides of each triangular projection are also of unit length.



A right triangle is identified having its longer leg coincident with one side of the hexagon plus the sides of two projections, which total three length units. In the same right triangle, the hypotenuse and shorter leg are coincident with the containing equilateral triangle and thus subtend an angle of $\pi/3$. The hypotenuse is therefore $(2/\sqrt{3})$ times the longer leg, or $2\sqrt{3}$ units; the shorter leg is half this length, or $\sqrt{3}$. The side of the containing triangle comprises one long and one short leg, which sum to $3\sqrt{3}$ units. The areas of the triangle and hexagon are then found using the formula for an equilateral triangle:

$$A_T = \frac{(3\sqrt{3})^2\sqrt{3}}{4} = \frac{27\sqrt{3}}{4} \qquad A_H = 6 \cdot \frac{(1)^2\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$$

$$\therefore \frac{A_H}{A_T} = \frac{(3\sqrt{3})/2}{27\sqrt{3}/4} = \frac{2}{9}$$

Thus the hexagon area is 2/9 that of the containing triangle. The full hexagram area is twice this, or 4/9.

Better late than never

J/F 2 Loren Bonderson notes that the first term in the final formula for *L* is missing a square root.

Other responders

R. Akkrut, F. Albisu, J. Bergman, S. Berkenblit, R. Bird, M. Bolotin, W. Chan, J. Chandler, T. Chow, F. Cogswell, G. Coram, A. Cosquer, R. Downes, J. Feil, T. Griffin, J. Grossman, J. Hardis, J. Larsen, A. LaVergne, Z. Levine, J. Mackro, F. Marcoline, N. Markovitz, Z. Master, T. Mattick, B. McQuain, T. Mita, G. Muldowney, J. Ng, A. Ornstein, F. Powsner, E. Prych, I. Resnikoff, B. Rhodes, P. Sanchez, H. Sard, A. Scharf, E. Schwabe, D. Sidney, W. Stein, Team Insπired, H. Terkanian, S. Vatcha, S. Wang, J. Winters, and K. Zeger.

Solution to speed problem

$$\frac{\sqrt{9}+\sqrt{9}}{\sqrt{9\%}}$$