Puzzle corner

For the first time in about a year, last Saturday we went to a restaurant! Alice and I and our companions have all been fully vaccinated; we all wore masks when not eating; the restaurant was well ventilated and had sanitizers on each table; we were seated far from other customers; and there were screens between tables. Nonetheless, it was a restaurant.

Problems

J/A1. George Fisher likes the card game euchre, where the deck contains the highest six cards of each suit. In late 2019, for the first time in his life, George was dealt a hand with all five cards in the same suit. He asks how the probability of being dealt five cards of only one suit with the euchre deck compares to the probability using a standard 52-card deck. He also wonders if there is a number of suits with the same expectation (say, within 1%) in both the euchre and the standard card deck. He also considers a minimalist deck with only two cards in each suit and wonders: With this deck, what is the probability of getting a hand with exactly one, two, three, or four suits?

J/A2. We now offer another of Richard Hess's "logical hat" problems. In each of these problems, logicians are each wearing a hat with a number. The logicians see the numbers on every other hat, but not the one on their own. Each logician is error-free in his or her reasoning and is given that information before the puzzle starts.

For this particular puzzle, there are three logicians, A, B, and C. All three know that each number is a positive integer and know that one of the numbers is the sum of the other two.

A announces, "I don't know my number," after which B announces, "My number is 10." What numbers are on A and C?

J/A3. William Stein offers us the following sobering puzzle.

The probability of a US antimissile missile hitting an incoming intercontinental ballistic missile (ICBM) is relatively low, typically between 0.5 and 0.6.

We call this probability a. Therefore, it is US doctrine to fire a "high number" m of antimissile missiles at each ICBM that we want to stop from landing in our territory. What is the probability of an ICBM getting through given a and m?

If we are attacked by N incoming ICBMs, what is the probability of at least one getting through given a, N, and m? (This requires $m \times N$ antimissile missiles.)

Speed department

Joseph Horton has noticed that it takes the sun about two minutes to set and asks, "What angle does the sun subtend from Earth?"

Solutions

M/A1. Larry Kells sent us a variation on 2020 M/J1. This time you hold the AKJ of spades, AKJ of hearts, AKQ of diamonds, and AKQJ of clubs. Can you always make 6 no-trump, or is there a lie of the cards where the opponents can beat you (with best play on both sides)? The following solution is from Len Schaider.

Declarer (South) can be stopped from making the 6NT contract. South has 11 sure high-card tricks but needs to win one more trick with either the J of spades or the J of hearts.

The distribution needed to defeat the contract is when one defender has at least four spades headed by the Q10 and the other defender has at least four hearts headed by the Q10; it does not matter if West has the spades and East has the hearts or vice versa. Distribution of the other cards in the West, North, and East hands is irrelevant. North (dummy) will not win any tricks, since South has all the high cards. Based on the distribution below, West would never discard a spade, nor would East discard a heart, unless that was their only choice. West must lead a diamond or club to avoid giving South a free finesse in spades or hearts.

South has three similar options for how to play the hand, but all of them fail. One follows; the complete solution is on the Puzzle Corner website.

South cashes 11 sure tricks (four clubs, three diamonds, two hearts, and two spades) and has the J of spades and J of hearts remaining. West has the Q10 of spades and East has the Q10 of hearts. South must lead a J, which will lose to the Q, and then the 10 of that same suit will win the trick to defeat the contract.

**M/A2.** Another "modest polyominoes" problem from Richard Hess and Robert Wainwright. You are to design a connected tile so that three of the tiles can cover at least 93% of the area of the shape shown. The tiles are identical in size and shape and may be turned over so that some are mirror images of the others. They must not overlap each other or the border of the polyomino.
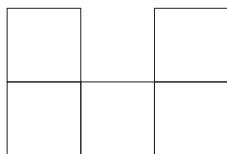
Eric Lachapelle sent us the following solution that gives 93.75% coverage.
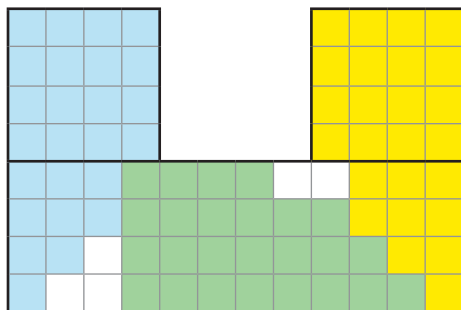
M/A3. Suppose a billiard ball is hit straight from the corner of an $a \times b$ rectangular billiard table at an angle of 60° as shown.
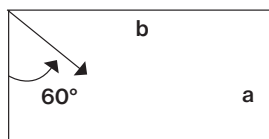
How far below the left bumper will the ball first strike the left cushion on its return?

Burgess Rhodes essentially "unfolded" the table (see diagram) and then all was well.

Insert the billiard table, with left cushion width a and left bumper length b, into the top left of the reference grids shown. The billiard ball, hit from upper-left corner H, travels a path which in the reference grids is a straight line, impacting the right side of the grid at point R'. If the grid is folded, first along the vertical midline M and then in "accordion" fashion back upon the billiard table, the straight line will trace the actual path of the ball to the point R of first return on the left cushion.

The required distance D below the left bumper to R is the distance on the right side of the reference grid between R' and the nearest H. Distance L on the right side of the reference grid from the top H down to R' is $L = 2b \times \tan 30°$.
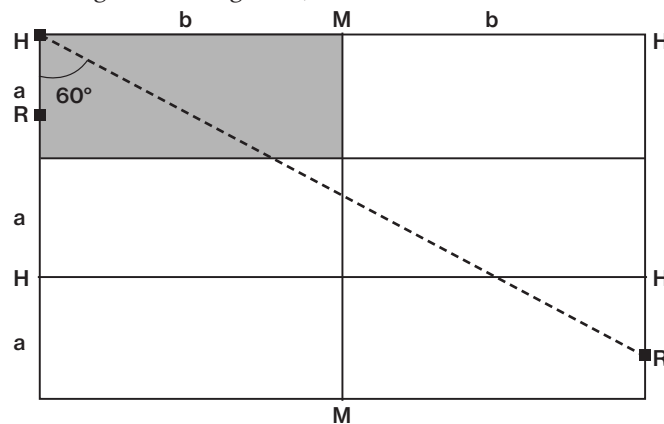
Let $K = \left\lfloor \frac{L}{a} \right\rfloor$. Then

$$D = \begin{cases} L - Ka & \text{if } K \text{ is even, and} \\ (K+1)a - L & \text{if } K \text{ is odd.} \end{cases}$$

In terms of the table dimensions: for $m = 0, 1, 2, \ldots,$

$$D = \begin{cases} \left(\frac{2b}{\sqrt{3}} - \left\lfloor \frac{2b}{a} \right\rfloor \right) a & \text{if } m\sqrt{3} \leq \frac{b}{a} < \left(m + \frac{1}{2}\right)\sqrt{3}, \text{ and} \\ \left(\left\lfloor \frac{2b}{a} \right\rfloor + 1 \right) a - \frac{2b}{\sqrt{3}} & \text{if } \left(m + \frac{1}{2}\right)\sqrt{3} \leq \frac{b}{a} < (m+1)\sqrt{3}. \end{cases}$$

Here is a diagram for $K = 2$, Even. See the Puzzle Corner website for a diagram showing $K = 3$, Odd.

Better late than never

N/D2. The solution to N/D2 addressed the determination of the coefficients for $\sum_{j=1}^{n} j^3$. Ed Koch tackled the more general question and reduced $\sum_{j=1}^{n} j^p$ to terms involving lower powers of j.

Other responders

Y. Aliyev, G. Angle, J. Bergmann, M. Bolotin, P. Cassady, W. Chan, J. Conway, F. Cornelius, T. Galaitsis, S. Gordon, J. Harmse, J. Iler, P. Kramer, J. Langer, J. Larsen, S. Lerman, S. Liu, J. Mackro, J. Marcou, T. Mattick, G. Muldowney, S. Nason, M. Perkins, E. Signorelli, S. Sperry, L. Stabile, G. Steele, U. Sukhatme, and D. Turek.

Solution to speed problem

$$\frac{2min}{24hr} = \frac{x}{360°}$$