# Puzzle corner

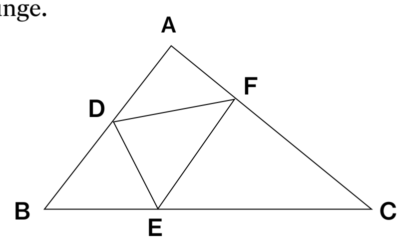
Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

Here's a reminder of Puzzle Corner's ground rules. Each issue contains three regular problems (one of which normally features bridge, chess, or another game) and one "speed" problem, whose solution appears at the end of the column. Readers submit solutions to the regular problems, and two issues later, one is printed for each. For example, solutions to problems below will appear in the September/October column, which I will submit in mid-June. Please try to send your solutions early. Late solutions and comments on published solutions are acknowledged in "Other Responders." Major corrections or additions to published solutions are sometimes printed in "Better Late Than Never," as are solutions to previously unsolved problems. An annual problem—and a solution to the previous year's problem—is published each January.

## Problems

**M/J1** Larry Kells wants you to find a legal chess game where White has all 16 starting men, Black has only his king, it is White's move, and White has been stalemated.

**M/J2** Tony Yen overheard the following problem in a Chinese airport lounge.




You are given that AD = DB, FC = 3AF, and EC = 2BE. Find the area of triangle ABC in terms of the area of triangle DEF.

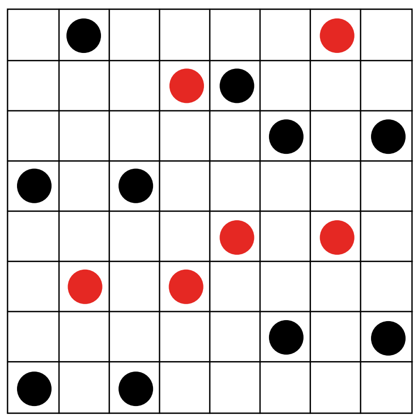
**M/J3** Sorab Vatcha has an equilateral triangle containing the largest regular hexagram that can fit inside. What is the ratio of the area of the hexagon to the area of the triangle?

## Speed department

Ermanno Signorelli wants you to arrange three 9s with standard arithmetic signs and symbols to yield the value 200.

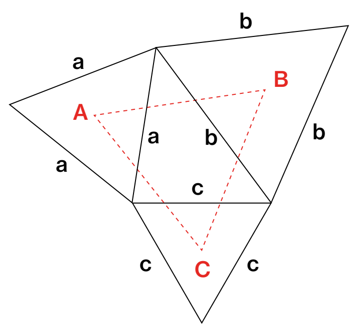
## Solutions

**J/F1** Ken Knowlton wondered what is the maximum number of checkers that can be placed on an 8 × 8 board so that no three checkers lie in a straight line (horizontal, vertical, or diagonal). As many responders noted, 16 is clearly an upper bound: if there are more than 16 checkers, then at least one of the eight rows must have more than two checkers. Stephen Dibert sent a photograph of a real checkerboard with 16 checkers meeting the required conditions. The following computer-drawn diagram shows Dibert's configuration.



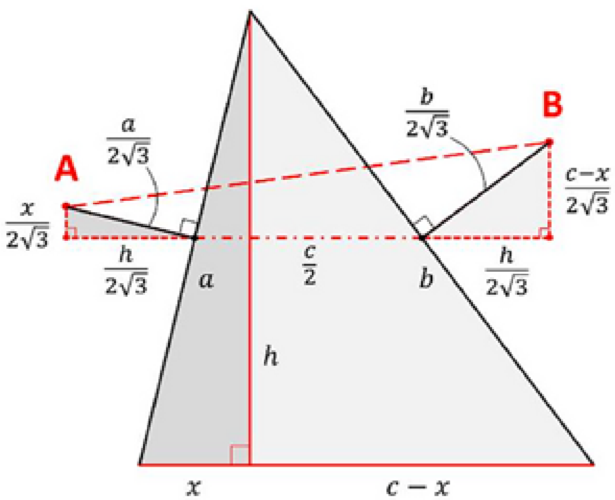
Burgess Rhodes sent, in addition, an example of a 19 × 19 board with 7 × 19 checkers: two in each row and column and in the two main diagonals.

**J/F2** Ermanno Signorelli wants you to show that Napoleon's triangle is always equilateral. For any triangle with sides *a*, *b*, and *c*, first draw equilateral triangles on each side. Let A be the center of triangle *aaa*. Do the same for B and C. Napoleon's triangle is then triangle ABC.



The following solution is from Greg Muldowney.

Considering side $AB$ in particular of Napoleon's triangle, we use side $c$ of the original triangle as a reference horizontal and construct the altitude $h$. The resulting segments of $c$, denoted $x$ and $(c - x)$, are the bases of two right triangles having hypotenuses $a$ and $b$ respectively. Points A and B, being centers of equilateral triangles $aaa$ and $bbb$, lie at perpendicular projections of lengths $a/2\sqrt{3}$ and $b/2\sqrt{3}$ from the midpoints of sides $a$ and $b$. The orthogonal components of each projection form a right triangle rotated 90° from, and necessarily similar to,




the corresponding triangle bounded by $c$ and $h$. The side lengths $\{x, h, a\}$ and $\{c - x, h, b\}$ are then directly scaled by $2\sqrt{3}$ to locate A and B. The orthogonal components of $AB$ are:

$$\Delta_{\text{Horizontal}} = \frac{h}{\sqrt{3}} + \frac{c}{2} \quad , \quad \Delta_{\text{Vertical}} = \frac{c-2x}{2\sqrt{3}}$$

The length $AB$ is then determined as follows:

$$(AB)^2 = \left(\frac{h}{\sqrt{3}} + \frac{c}{2}\right)^2 + \left(\frac{c-2x}{2\sqrt{3}}\right)^2 = \frac{hc}{\sqrt{3}} + \frac{h^2+c^2-cx+x^2}{3}$$

The second term is simplified using Pythagorean relations in the triangles formed by $h$:

$$h^2 = a^2 - x^2 = b^2 - (c-x)^2 \quad \rightarrow \quad \therefore\ cx = \tfrac{1}{2}(a^2 - b^2 + c^2)$$

$$h^2 + c^2 - cx + x^2 = a^2 + c^2 - \tfrac{1}{2}(a^2 - b^2 + c^2)$$
$$= \tfrac{1}{2}(a^2 + b^2 + c^2)$$

Side $AB$ of Napoleon's triangle thus has the length:

$$AB = \sqrt{\frac{hc}{\sqrt{3}} + \frac{a^2+b^2+c^2}{6}}$$

The product of base and altitude $hc$ is twice the triangle area—a constant for $abc$ unrelated to the Napoleon triangle construction. The sum of squares is also unchanged by which side of Napoleon's triangle (and side $a$, $b$, or $c$ as reference line) is examined. It follows that the above length expression describes all three sides of Napoleon's triangle.

Substituting Heron's formula for the triangle area in terms of sides $a$, $b$, and $c$ with $s = (a + b + c)/2$ yields:

$$L = \sqrt{\frac{(a+b+c)(s-a)(s-b)(s-c)}{\sqrt{3}} + \frac{a^2+b^2+c^2}{6}}$$

as the side length of Napoleon's triangle constructed around a triangle of sides $a$, $b$, and $c$.

## Better late than never

**Y2020** John Chandler notes three improvements—$1 = 202^0$, $2 = 2 + 0^{20}$, $3 = 2 + 20^0$—and one correction: $23 = 22 + 0^0$ (also noted by Alan Levin, Tom Gauss, and Steve Silberberg). Levin and Gauss also found $40 = 20 + 20$, which keeps the digits in order. Chandler believes that solutions look neater with exponentiation than other operators and suggests, all else being equal, that I prefer those solutions. I shall try to remember to do so next October when I prepare the January/February column.

## Other responders

M. Bolotin, L. Bonderson, M. Brunschwig, S. Bryant, J. Feil, C. Gabor, J. Grossman, J. Langer, J. Larsen, P. Linsay, R. Lipes, J. Macro, T. Mattick, T. Mita, G. Muldowney, J. Pari, E. and C. Passow, D. Puppin, B. Rhodes, K. Rosato, D. Sidney, S. Spitz, D. Stork, U. Sukhatme, H. Thiriez, P. Winterfeld, A. Yen, and T. Yen.

## Solution to speed problem

(9 + 9)/9%