Puzzle corner

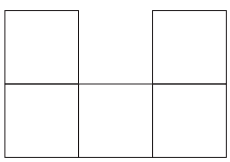
Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

Tom Harriman, a regular contributor to Puzzle Corner, has become interested in a variant of Fermat's last theorem, specifically integral solutions to $a^2 + b^3 = c^4$. Anyone wishing to join the effort should contact Tom directly at tiotom@cox.net.

Problems

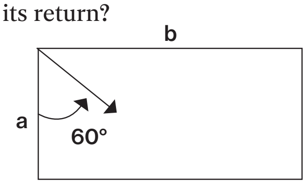
M/A1. Larry Kells sends us the following variation on 2020 M/J1. This time you hold AKJ of spades, AKJ of hearts, AKQ of diamonds, and AKQJ of clubs. Can you always make 6 no-trump or is there a lie of the cards where the opponents can beat you (with best play on both sides)?

M/A2. Another "modest polyominoes" problem from Richard Hess and Robert Wainwright. You are to design a connected tile so that three of the tiles can cover at least 93% of the area of the shape below. The tiles are identical in size and shape and may be turned over so that some are mirror images of the others. They must not overlap each other or the border of the polyomino.



M/A3. Suppose a billiard ball is hit straight from the corner of an $a \times b$ rectangular billiard table at an angle of 60° as shown.

How far below the left bumper will the ball first strike the left cushion on its return?



Speed department

Ken Knowlton asks if the number of times between noon and midnight that a clock's minute hand passes its hour hand is half the number between successive midnights.

Solutions

N/D1. Jorgen Harmse sent us the following, based on an actual deal.

The contract should have been 7 hearts (unless you were warned about the clubs), but you're in 6 no-trump. West leads the jack of clubs, and you immediately win two club tricks. What are your chances of making the contract?

	♠ 52 ♥ A843 ♦ A9654 ♣ 86	
♠ ?? ♥ ?? ♦ ?? ♣ Jt532		♠ ?3 ♥ ?? ♦ ?? ♣ 7
	♠ AKQ ♥ KQt6 ♦ K ♣ AKQ94	

I received several fine solutions but lack the expertise to evaluate them comprehensively. I found Len Schaider's convincing and chose it for publication but must mention Frank Model's comment that the opening lead was not well chosen by the defense. Schaider's solution:

The contract of 6 no-trump can be made. South has 11 sure tricks: three spades, three hearts, two diamonds, and three clubs. The challenge is to win either a fourth club or a fourth heart to end up with 12 tricks. Based on the play of the first two tricks, South knows the club distribution in the E/W hands but has no idea about the other three suits in those hands. South must determine where the hearts are. At trick 3, South proceeds as follows:

a. Cash the king of diamonds.

b. Cash the king of hearts.

c. Lead a heart to the ace of hearts.

d. Cash the ace of diamonds (sixth trick) and discard a club from the South hand. At this point, South will know if West started with no hearts or only one heart. If either of these is true, East was dealt four or five hearts. South would lead a low heart and finesse with the Q10 in his hand and could then cash the fourth heart trick, three spades, and the queen of clubs to have 12 tricks.

e. If both East and West played a heart on the first two heart leads, South knows the heart distribution is 2/3 or 3/2 in the E/W hands. In this case, there is only one heart outstanding and South leads a low heart to the queen, cashes the 10, and has four heart tricks. South would then cash three high spades and the queen of clubs to have 12 tricks.

f. If West has four or five hearts, then South would change plans and lead a spade from dummy and cash three spade tricks, giving a total of nine tricks. At this point, West and South have four cards. (West: hearts J 9; clubs 10 5. South: hearts Q 10; clubs Q 9.)

South has two choices, each with the same result: 1) Cash the Q of hearts and play the 10 of hearts, giving the lead to West, who must play a club, giving South a free finesse and the last two tricks. 2) Cash the Q of clubs and play the 9 of clubs, giving the lead to West, who must play a heart, giving South a free finesse and the last two tricks.

g. West could try to fool South by not keeping two hearts and/or two clubs and end up with something like: hearts J 9, club J, and a spade or diamond. South would be counting cards and know if this is the situation and would cash the Q and 10 of clubs and the Q of hearts to win 12 tricks.

N/D2. David Mayhew has been given the task of evaluating the sequence $1^3 + 2^3 + 3^3 + \ldots + n^3$ for large n and without computing hardware. A straightforward term-by-term calculation seems virtually impossible to accomplish in a lifetime. Prove that the formula $c_1 n + c_2 n^2 + c_3 n^3 + c_4 n^4$ will evaluate the sequence for all n and find the coefficients that make it happen.

Alice Deanin notes that the sum of the first n cubes is equal to the square of the nth triangular number. That is,

$$\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{1}{4}n^2 + \frac{1}{2}n^3 + \frac{1}{4}n^4$$

which is a formula of the correct form. That it works for all $n > 0$ is proved by induction as follows. It is clearly true for $n = 1$. For $n + 1$, the formula yields

$$\frac{1}{4}(n+1)^2 + \frac{1}{2}(n+1)^3 + \frac{1}{4}(n+1)^4 =$$

$$\frac{1}{4}(n^2 + 2n + 1) + \frac{1}{2}(n^3 + 3n^2 + 3n + 1) + \frac{1}{4}(n^4 + 4n^3 + 6n^2 + 4n + 1) =$$

$$\frac{1}{4}n^2 + \frac{1}{2}n^3 + \frac{1}{4}n^4 + \left\{\frac{2n+1}{4} + \frac{3n^2 + 3n + 1}{2} + \frac{4n^3 + 6n^2 + 4n + 1}{4}\right\} =$$

$$\frac{1}{4}n^2 + \frac{1}{2}n^3 + \frac{1}{4}n^4 + \{n^3 + 3n^2 + 3n + 1\} =$$

$$\frac{1}{4}n^2 + \frac{1}{2}n^3 + \frac{1}{4}n^4 + \{(n+1)^3\}$$

N/D3. Richard J. Morgen offers a math problem that stems from a TV quiz show in which three contestants get to spin a wheel. Each contestant can choose to spin once or twice. The wheel has 20 equal-size segments, marked 5, 10, 15, 20, on up to 100. Each contestant can decide, in turn, whether to stop after one spin or spin again. Anyone who goes over 100 is out. A tie (which could be two-way or three-way) results in a one-spin spinoff.

What is the best strategy? Specifically, what number should you stop on if you spin first? What if you spin second? (Obviously, if you are second, you take a second spin if your first spin does not match or exceed that achieved by the first spinner. But you need more strategy than that.) The third spinner only has to decide whether to stop if he or she ties someone.

In addition to sending his magnum opus, which he calls "too long for e-mail or print," Jonathan Hardis sent the following summary (naturally, the full solution has a permanent home on the Puzzle Corner website).

The first spinner should take a second spin if the first number is 65 or less. On best play, the overall odds of winning are 30.82% if this is done.

The second spinner should take a second spin if either 1) the first number is 50 or less, 2) the first number is less than the result from the first spinner, or 3) the first number ties the first spinner with that number being 65 or less. On best play, the overall odds of winning are 32.96% if this is done.

The third spinner should take a second spin if either 1) the first number is less than value needed to win, 2) the first spin ties both preceding spinners, if the tie is at 65 or less, or 3) the first spin ties only one previous spinner, if the tie is at 50 or less. (More precisely, if tied with only one previous spinner at 50, it makes no difference if a second spin is taken or not. The odds of winning are 50/50 from that point either way.) On best play, the overall odds of winning are 36.22% if this is done.

Better late than never

N/D SD. Jie Wu and Kevin Koch note that all the originals are Pythagorean triplets (without removing one digit). David Saslav adds that this requires changing the third set slightly. Jerald Baronofsky noted another oddity: the first number squared is the midpoint of the remaining two numbers.

Other responders

S. Alexander, J. Bergmann, W. Bishop, S. Blumsack, M. Bolotin, L. Bonderson, W. Chan, D. Detlefs, J. Ebert, D. Fitterman, S. Frymer, J. Grossman, J. Hardis, L. Kaatz, E. Kaplan, J. Karlsson, E. Kutin, J. Langer, J. Larsen, K. Lebensold, J. Lefferts, A. Levine, S. Liu, D. Loeb, R. Loretz, W. Ludington, P. Manglis, N. Markovitz, C. Marks, R. Marks, J. Marlin, J. Martin, D. Mellinger, T. Mita, S. Nason, H. Nicholson, J. Orwant, T. Royappa, J. Russell, B. Snyder, S. Sperry, K. Swartz, D. Turek, S. Ulens, B. Wake, C. Wampler, A. Wiegner, and D. Wiley.

Solution to speed problem

No, it is 10 versus 21.