Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

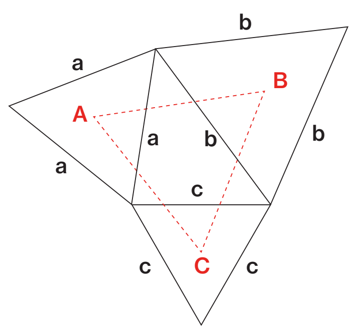
This being the first issue of a calendar year, we again offer a "yearly problem" as well as the solution to last year's problem.

Problems

Y2021 How many integers from 1 to 100 can you form using the digits 2, 0, 2, and 1 exactly once each, along with the operators +, −, × (multiplication), / (division), and exponentiation? We desire solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 2, 1 are preferred. Parentheses may be used; they do not count as operators. A leading minus sign, however, does count as an operator.

J/F1 Ken Knowlton wonders what is the maximum number of checkers that can be placed on an 8 × 8 board such that no three checkers lie in a straight line (horizontal, vertical, or diagonal).

J/F2 Ermano Signorelli wants you to show that Napoleon's triangle is always equilateral. For any triangle with sides *a*, *b* and *c*, first draw equilateral triangles on each side. Let A be the center of triangle *aaa*. Do the same for B and C. Napoleon's triangle is then triangle ABC.



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Speed department

Sorab Vatcha wants to know the longest English word with its letters in reverse alphabetical order. Repeated consecutive letters, such as "oo" in "wool," are permitted.

Solutions

Y2020 As expected, the year 2020 was bad for our yearly problem. Unexpectedly, it was even worse for humankind's health. Although I do not like 0^0, I decided to permit it this time and even allowed Bob Anderson's exotic use for 8 in his solution below:

$1 = 2^0 + 2 \times 0$	$8 = 2^{2+0^0}$	$21 = 20 + 2^0$
$2 = 2^0 + 2^0$	$9 = (2 + 0^0)^2$	$22 = 20 + 2 + 0$
$3 = 2^0 + 2 + 0$	$10 = 20/2 + 0$	$23 = 22 + 2^0$
$4 = 2 + 0 + 2 + 0$	$18 = 20 - 2 + 0$	$40 = 20 \times 2 + 0$
$5 = 2^2 + 0^0$	$19 = 20 - 2^0$	$100 = 200/2$
$6 = (2 + 0^0) \times 2$	$20 = 20 \times 2^0$	

S/O1 In Richard Thorton's bridge problem, Dick Overbid has bid 7 no-trump and his partner, Jane Mathwhiz, is aghast when she sees Dick's hand. Between them they have all the aces, queens, and jacks, but no kings. To make the contract, she must make two successful finesses. She can finess East for both red kings or West for both black kings, or some combination thereof. Jane immediately chooses the play with maximal success probability. Before any card was played, what was her probability of success?

The following solution is from Jim Larson.

Jane must make all the tricks to complete the 7 no-trump contract, so the specification that she must make two successful finesses implies that she can win the other two kings by the only other method available, dropping singleton kings with her aces.

The following diagram gives a suggestion of what portions of the N-S hands might look like.

	North	
	♠ A Q J 10 ♥ x x x x ♦ x x x x ♣ A Q J 10	
♠ ♥ ♦ ♣		♠ ♥ ♦ ♣
	♠ x x x x ♥ A Q J 10 ♦ A Q J 10 ♣ x x x x	

Since she is able to make the finesses specified, she will have aces behind the targeted kings and enough length in those suits to finesse repeatedly without the opponent outlasting her.

A decent strategy is for Jane to choose the two kings that can't be dropped, out of the four choices available, and finesse for those. The resulting probability of Jane being successful with this strategy is: 1/6 × ½ × ½ = 1/24.

However, Jane has a better strategy available. She should finesse all four suits. The location of each of the kings is independent, so the chance of all kings being on the correct side is (½)4 = 1/16, which is a higher probability.

S/O2 Rather then having soldiers march in a square, Phil Cassady has a square of marching soldiers. He writes:

As a square group of soldiers begins marching forward in formation, their mascot dog starts running forward from the back edge along the center line of the square formation. When he reaches the front edge, he immediately reverses his run along the center line; he reaches the rear edge of the formation just as it passes the starting location of the front edge. If the square formation is 50 feet on a side, how far does the dog run?

The following solution is from Robert Bird.

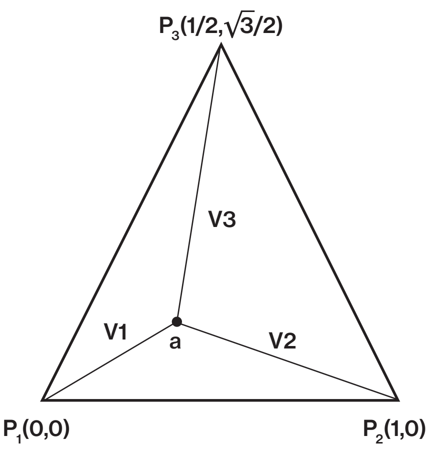
The leading edge of the group of soldiers travels a distance, x, at velocity, V, in the time, t, required for the dog to reach at velocity v. (1) $x = Vt$ and (2) $50 + x = vt$. The trailing edge of the group travels 50 feet in the time, T, for the dog to reach them: (3) $50 = VT$ and (4) $50 + 2x = vT$.

Dividing (2) by (1) gives $(50 + x)/x = v/V$ and dividing (4) by (3) gives $(50 + 2x)/50 = v/V$. Hence $(50 + x)/x = (50 + 2x)/50$, which gives $x = 35.355$ feet. So the dog travels $50 + 2x = 120.71$ feet.

S/O3 We end with a different kind of geometry problem from Bruce Heflinger—one where none of the parts are moving.

Three line segments whose lengths satisfy $a^2 + b^2 = c^2$ meet at a point (x_0, y_0) interior to an equilateral triangle, each segment having its opposite end at a vertex of the triangle. Show that the angle between the two shorter segments is 150°.

Phil Winterfeld sent the following solution, which makes the problem appear much less ferocious than I had feared.



Consider the equilateral triangle shown in the preceding figure with vertices P_1, P_2, and P_3 located at points (0,0), (1,0), and (1/2, $\sqrt{3}/2$), respectively. The point of interest, (X,Y), is located inside the triangle. Form vectors $\vec{v}_i$ by connecting points P_i to (X,Y), where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors in the x and y directions, respectively:

$$\vec{v}_1 = -X\hat{\imath} - Y\hat{\jmath}$$

$$\vec{v}_2 = (1 - X)\hat{\imath} - Y\hat{\jmath}$$

$$\vec{v}_3 = \left(\tfrac{1}{2} - X\right)\hat{\imath} + \left(\tfrac{\sqrt{3}}{2} - Y\right)\hat{\jmath}$$

The point (X,Y) is chosen such that:

$$|\vec{v}_1|^2 + |\vec{v}_2|^2 = |\vec{v}_3|^2$$

Show that the angle, a, between $\vec{v}_1$ and $\vec{v}_2$ is 150°.

The angle a is given by the definition of the dot product between two vectors:

$$cos(a) = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1||\vec{v}_2|}$$

$$cos(a) = \frac{-X(1-X)+Y^2}{\sqrt{X^2+Y^2}\sqrt{(1-X)^2+Y^2}}$$

The relation between the vectors' magnitudes is:

$$X^2 + Y^2 + (1 - X)^2 + Y^2 = \left(\tfrac{1}{2} - X\right)^2 + \left(\tfrac{\sqrt{3}}{2} - Y\right)^2$$

This reduces to:

$$X^2 + Y^2 = X - \sqrt{3}Y$$

Then,

$$cos(a) = \frac{-X+X^2+Y^2}{\sqrt{X^2+Y^2}\sqrt{X^2-2X+1+Y^2}}$$

$$cos(a) = \frac{-\sqrt{3}Y}{\sqrt{X-\sqrt{3}Y}\sqrt{-\sqrt{3}Y-X+1}} = \frac{-\sqrt{3}}{2}$$

$$a = cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = 150°$$

Other responders

A. Andersson, L. B., T. Barrows, J. Bergmann, M. Brand, W. Burke, J. Chandler, B. Deitrick, N. Derby, S. Dibert, J. Feil, G. Fischer, S. Golson, T. Griffin, J. Hardis, T. Harriman, A. Hirshberg, J. Kotelly, P. Kramer, N. Lang, J. Langer, R. Lipes, J. Mackro, M. Marinan, T. Mattick, S. McGinnis, D. Mellinger, A. Moulton, G. Muldowney, S. Nason, A. Ornstein, R. Orr, J. Prussing, B. Rhodes, J. Rulnick, L. Schaider, B. Schargel, S. Silberberg, T. Sim, J. Steel, A. Stern, S. Swaminathan, M. Viswanathan, D. Wagger, R. Wake, R. Whitman, J. Winters, and Y. Zuss.

Solution to speed problem

Spoonfeed