Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

I have sad news to report: my mother-in-law, Eva Schenk, died on May 17. Eva was a Holocaust survivor whose husband died when their twins were just 10 weeks old. As my father died when I was in high school and my mother died when I was in graduate school, Eva has been my parent figure for longer than my natural parents combined. She will be missed, and I dedicate this column to her memory.

Problems

S/O1. We begin with a bridge problem from Richard Thornton:

Dick Overbid has bid 7 no-trump, and his partner, Jane Mathwhiz, is aghast when she sees Dick's hand. Between them they have all the aces, queens, and jacks, but no kings. To make the contract she must make two successful finesses. She can finesse East for both red kings or West for both black kings, or some combination thereof. Jane immediately chose the play with maximal success probability. Before any card was played, what was her probability of success?

S/O2. Rather than having soldiers march in a square, Phil Cassady has a square of marching soldiers. He writes:

As a square group of soldiers begins marching forward in formation, their mascot dog runs at a constant rate forward from the back edge along the centerline of the square formation. When he reaches the front edge, he immediately reverses his run along the centerline; he reaches the rear edge of the formation just as it passes the starting location of the front edge. If the square formation is 50 feet on a side, how far does the dog run?

S/O3. We end with a different kind of geometry problem from Bruce Heflinger, one where none of the parts are moving.

Three line segments whose lengths satisfy $a^2 + b^2 = c^2$ meet at a point (x_0, y_0) interior to an equilateral triangle, each segment having its opposite end at a vertex of the triangle. Show that the angle between the two shorter segments is 150°.

Speed department

SD. Avi Ornstein advises extra caution at 7:06 on August 5. Why?

Solutions

M/J1. We begin with a bridge problem from Larry Kells. Assume you hold

Spades: AQ10
Hearts: AKQ
Diamonds: AKQ
Clubs: AKQJ

You bid 6 no-trump and play it there. Can you make this contract against any distribution of cards to the remaining three hands, assuming best play on all sides? You should also assume that you and the opponents know the distribution.

Jorgen Harmse sent us the following solution.

Six no-trump is easy if dummy has the king or jack of spades or the opening lead is a spade. If not, I make 12 tricks by a squeeze without the count as follows:

Suppose that Player A has the king of spades. I dare not give up my stopper in any suit of which Player A has more cards than I do, but I can safely run any side suit once Player A has the same number or fewer. There is such a suit as long as Player A has three or more spades and at least one non-spade, so I reach an end position in which either Player A has fewer than three spades and I can win any return from Player A, or Player A has nothing but spades. In the former case I cash the ace and use the 10 to pull down the king, unless Player A dumps the king (in an attempt to give the lead to my other opponent), in which case I cash the queen and run the remaining side-suit winners. In the latter case, the sequel depends on who has the jack: if Player A has the jack then I throw the lead (or win cheaply) by playing the 10, but if the other opponent has the jack then I finish running side suits and see what that opponent keeps.

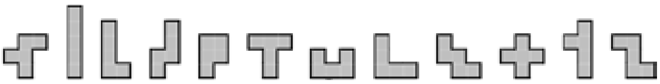
Consider an end position in which I have run the side suits, Player A still has Kxx in spades, and the other defender has the jack. (Dumping K or J doesn't help the defense.) If that defender still has a side-suit card, then the jack is doubleton or singleton, so I capture it by playing the ace and queen. (Player A can win the king, but then I win the 10.) If instead that defender has Jxx, then I throw the lead by playing the card that my left-hand opponent can barely beat. Either my opponents give up a trick immediately (for example, by using the king to beat the 10) or the return lead comes from my left and gives me a free finesse.

M/J2. Our second offering is an unusual pentomino problem from H. Yamamoto (via that most prolific puzzler Nob Yoshigahara). First choose one of the 12 pentominoes as your tile. Recall that a pentomino contains five 1×1 squares, and consider an 8×8 board, initially empty. What is the maximum number of your tiles that can be placed on the board without overlap? The answer depends on your choice of tile, so a full solution consists of determining the maximum for each of the 12 distinct pentominoes.

I received several beautiful solutions; Michael Branicky's was able to pack in the most pentominoes. His words are below. His full solution appears on the Puzzle Corner website.

The objective is to pick a pentomino tile so as to maximize the number of squares that can be covered with copies of it on an 8×8 board without overlap. For each pentomino tile shape, we will find the maximum number of tiles that can be placed.

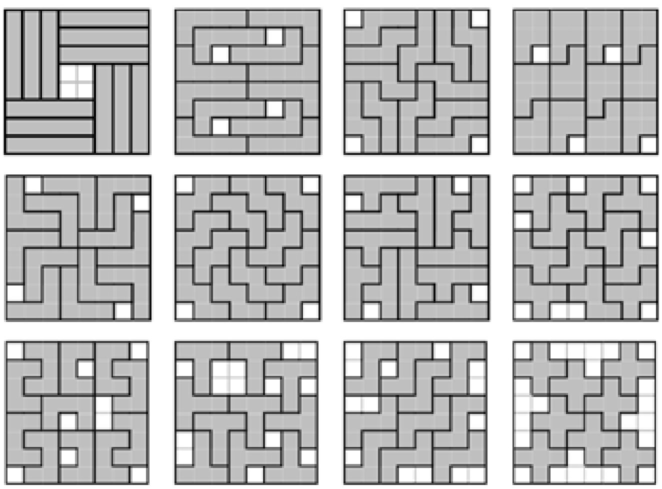
We'll name the 12 tiles F, I, L, N, P, T, U, V, W, X, Y, and Z:



Since each pentomino consists of five squares, the optimal number of tiles that can ever be placed is $\lfloor 64/5 \rfloor = 12$.

Optimal coverings can be achieved by placing 12 copies of the I-tile, L-tile, N-tile, P-tile, V-tile, W-tile, and Y-tile. The sorted maximal coverings for each pentomino are as follows:

pentomino shape	I	L	N	P	V	W	Y	F	U	T	Z	X
max. tiles placed	12	12	12	12	12	12	12	11	11	10	10	8



Some solutions could be found, or tuned, by hand. However, the solutions above and verification of their maximality came from computer search. All problems were solved by exhaustive search, using backtracking combined with transposition tables (i.e., caching of results). At each point, every placement of a given tile plus all its (unique) reflections and rotations was considered.

M/J3. Bruce Heflinger has a question about the terms in the Fibonacci sequence, which begins 1, 1, 2, 3, 5 . . . and is defined by the equations $F(0) = F(1) = 1$ and $n >= 2$, $F(n) = F(n\ 1) + F(n\ 2)$. Heflinger asks you to show that any two adjacent numbers in the sequence are relatively prime (i.e., they share no common factor other than 1).

Most responders used an inductive proof. Dave Mellinger ran his backwards, giving the following proof by contradiction:

Suppose two numbers in the sequence $F(n)$ and $F(n+1)$ are *not* relatively prime, so they share a common factor greater than 1—call it q. Then

$F(n) = qa$ and $F(n + 1) = qb$

We know that $F(n - 1) + F(n) = F(n + 1)$ or

$F(n - 1) = F(n + 1) - F(n)$

$= qb - qa$

$= q(b - a)$

Thus the next smaller Fibonacci number is also divisible by q. And so is the next smaller one, and the next smaller one, and ... Eventually we reach the small Fibonacci numbers, where we encounter $F(2) = 2$ and $F(3) = 3$. Their largest common factor is 1, meaning they do not have a common factor of q as required.

This contradiction shows that the original numbers $F(n)$ and $F(n + 1)$ must be relatively prime.

Better late than never

M/A SD and **M/J SD.** Many readers noticed that my wording permitted many solutions. I should have required *nonzero* integers. Mark Kantrowitz found an alternate solution to the corrected **M/A SD:** $8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 = 63 \times 64 \times 65 \times 66 = 17,297,280$. Dale Worley refers us to mathoverflow.net "equal products of consecutive integers."

Other responders

M. Barr, B. Bramley, R. Bumby, J. Chandler, I. Chatzigiannelis, D. Dechman, G. Fischer, J. Freilich, S. Golson, E. Gow, J. Grossman, P. Hartman, A. Hochbaum, C. Jackson, M. Kantrowitz, N. Lang, J. Langer, J. Larsen, S. Lerman, T. Mattick, T. Mita, G. Muldowney, P. Neirinckx, B. Rhodes, B. Schaefer, L. Schaider, E. Signorelli, T. Sim, S. Simanca, G. Steele, D. Worley, and J. Wrinn.

Proposer's solution to speed problem

666 is the Number of the Beast, signifying lawlessness. 6:66 is 66 minutes after 6, or 7:06. 6/66 is the 66th day of June, which is August 5.