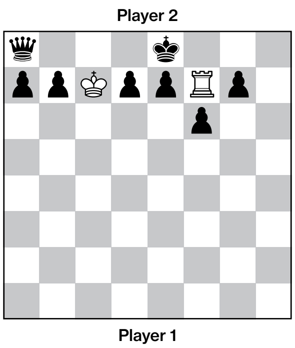
Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

It is time to review when and how I choose solutions for publication. The "when" is easy: the middle of the second month of publication. For example, these solutions to N/D problems were chosen in mid-December. The "how" is more difficult: the most important criteria are correctness and clarity. I favor electronic solutions, since they simplify typesetting, and solutions from first-time responders. Also considered is the space needed. I do not consider the date of submission.

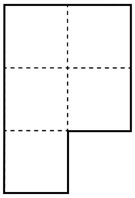
Problems

M/A1. We start with another "ternary reasoning" chess problem from Timothy Chow.



As usual for these problems, the question is whether the position can be reached in a legal chess game, and if so, whose turn it is.

M/A2. We continue with a "modest polyomino" problem from Richard Hess and Robert Wainwright. Design a connected tile so that six of them cover at least 93% of the area of the pentomino given below. The tiles are identical in size and shape and may be turned over (so that some are mirror images of the others). They must not overlap each other or the border of the pentomino.



M/A3. Our last regular problem is a venture into astronomy/3D geometry from Lee Giesecke.

Imagine it's the winter solstice and you want to compute the geocentric and geodetic latitudes of the Arctic Circle and the Tropic of Capricorn. The geocentric latitude gives the angle between the equatorial plane and a line from Earth's center to a point on the surface. The geodetic latitude assumes a line from the same surface point that is perpendicular to a plane tangent to Earth's surface. The angle of intersection of this line with the equatorial plane gives the geodetic latitude. Assume the Earth can be represented by an ellipsoid of revolution with the semi-major and semi-minor axes $a = 1$ and $b = 0.99665$. Assume that Earth's obliquity (axial tilt with respect to the plane of the ecliptic) is 23.44°.

Speed department

SD. Sorab Vatcha wants you to find seven consecutive integers whose product equals the product of four consecutive integers.

Solutions

N/D1. Our "games" problem this month, from Duncan Sheldon, is actually a combinatorics problem relevant to arranging an evening of bridge.

A bridge club consists of eight men and eight women who play at four tables. If the players are assigned so that all possible arrangements are equally likely, which of the following arrangements is more likely?

1. All partners are opposite sex.
2. All partners are same sex.

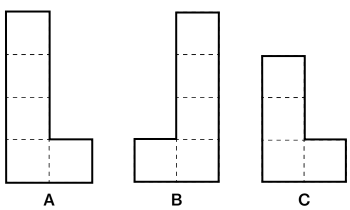
J. Light gives us a simple qualitative argument why the opposite-sex case is more likely, as well as a quantitative analysis showing the exact increase in probability. He writes:

It is more likely that all partners are opposite sex. Intuitively, this should be our first guess; after all, if I am in this club, I have eight possible partners of the opposite sex, and only seven of the same sex. Mathematically, we can work out the number of ways to assign teams as follows:

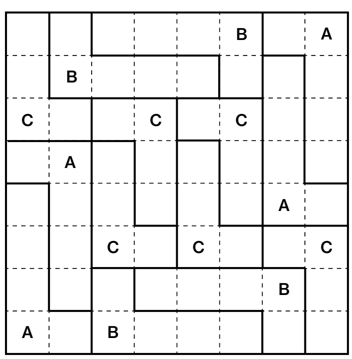
1. The number of ways to make opposite-sex teams is 8! = 40, 320. To see this, line up the women in alphabetical order. Each possible ordering of the men then gives one possible partnership; reordering the women gives no new combinations.

2. To make same-sex teams, line up the men by height. The tallest man has seven choices of partner. The tallest remaining man has five choices, the next three, and the final pairing is determined. The number of possible male teams is then 7 x 5 x 3 = 105. The women will independently have an equal number of parings, giving us only $1{,}05^2$ = 11,025 possible arrangements of same-sex teams.

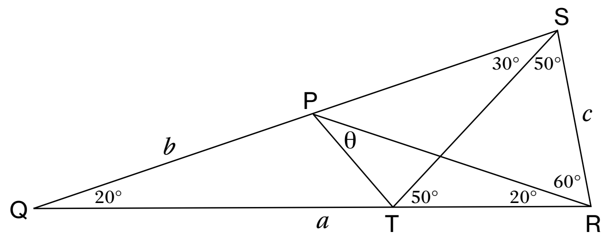
N/D2. Nob Yoshigahara offers the following tiling problem from Osho in Kyoto. You are to construct an 8 x 8 square from four copies of A, four copies of B, and six copies of C. You may rotate the shapes but cannot flip them over.



Richard Leuba (age 92!) sent us the following solution, about which he asserts, “Success with my 13th attempt!”



N/D3. Our last regular problem is a geometry offering from Bruce Heflinger: In the diagram that follows, PQR is a 140-20-20 isosceles triangle, with leg QP extended as shown. A line segment is constructed from point R, 60° clockwise from leg RP, to meet the extension of QP at point S. A second line segment is constructed from point S, 50° clockwise from SR, intersecting QR at point T. A third line segment is constructed from point T to point P. Evaluate angle TPR (marked as θ in the diagram).



The following solution is from Anthony Yen, whose diagram appears above. The only differences from the diagram given in the problem are the use of θ, noting that $\angle QST = 30°$ since $\angle Q + \angle R + \angle S = 180°$, and the addition of the labels $a = QR = QS$, $b = QP = RP$, and $c = RS = TR$. Yen then proceeds as follows:

Since $\triangle PQR$ is isosceles, we have, using the law of cosines

$$a^2 = 2b^2 - 2b^2 \cos 140° = 2b(1 + \cos 40°) = 4b^2 \cos^2 20°$$

Taking the square root and dividing by b gives $a/b = 2 \cos 20°$.

Since $\triangle RQS$ is also isosceles,

$$c^2 = 2a^2 - 2a^2 \cos 20° = 2a^2(1 - \cos 20°) = 4a^2 \sin^2 10°$$

Taking the square root gives $c = 2a \sin 10°$ or $a/c = 1/(2 \sin 10°) = \sin 30°/ \sin 10°$

Using the identity

$$\sin a - \sin \beta = 2 \cos \frac{a + \beta}{2} \sin \frac{a - \beta}{2}$$

We see

$$\frac{(a - c)}{c} = \frac{(\sin 30° - \sin 10°)}{\sin 10°} = \frac{2 \cos 20° \sin 10°}{\sin 10°} = 2 \cos 20°.$$

Therefore, $a/b = (a - c)/b$; which shows that $\triangle PRT$ is similar to $\triangle SQT$ and hence $\theta = \angle QST = 30°$.

Other responders

B. Adams, S. Avrutin, M. Barr, E. Bassan, J. Bergmann, E. Berlin, R. Bird, M. Branicky, P. Cassady, M. Chartier, J. Cooper, P. Davis, J. & J. Demers, F. Dippolito, J. Feil, G. Fischer, J.-P. Garric, D. Goldfarb, J. Grossman, C. Hibbert, H. Hodara, P. Kramer, J. Larsen, W. Lemnios, L. Lerman, Z. Levine, D. Linden, J. Mackro, T. Maloney, N. Markovitz, D. Mellinger, Z. Mester, R. Morgen, S. Nason, J. Norvik, R. Orr, E. Passow, S. Raheja, B. Rhodes, A. Ritter, D. Sapan, D. Sheldon, E. Signorelli, J. Sinnett, P. Steven, M. Strauss, N. Terraz, W. Thoen, S. Ulens, D. Wiley, and J. Williams.

Solution to speed problem

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 7 \times 8 \times 9 \times 10 = 5{,}040$