Puzzle corner

This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 2, and 0) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2019 yearly problem is in the "Solutions" section.

For regular problems, I choose solutions for publication in the middle of the second month that the problems appeared. For example, I selected the S/O solutions for this column during mid-October. No preference is given to solutions that arrive earlier. Please note that I prefer solutions that are neatly written, typed, or (especially) sent electronically, since they simplify typesetting.

Problems

Y2020. How many integers from 1 to 100 can you form using the digits 2, 0, 2, and 0 exactly once each, along with the operators +, −, ×, ÷, and exponentiation? We desire solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 2, 0 are preferred. Parentheses may be used; they do not count as operators. A leading minus sign, however, does count as an operator. Moreover, 0^0 evaluates to 1.

Your editor fears that the digits in the next few years will offer slim pickings for the yearly problem.

J/F1. I misinterpreted 2019 M/J1 (see "Better late than never") and so reopen the problem, with the correct interpretation, as J/F1.

Larry Kells wants you to construct a single full deal (i.e., specify all four hands) where, with South as declarer, the opponents can defeat every possible contract—and to maximize the number of high-card points South can hold in such a deal. To be clear, with this one full deal any contract by South can be defeated with best play on both sides.

J/F2. Richard Thornton sometimes overpays, since he occasionally multiplies the costs of individual items instead of summing them. (We assume all items cost a positive integral multiple of cents.) One time, he purchased four items whose total cost is $7.11, but he was lucky since the product was also $7.11. What did the individual items cost?

Thornton also asks a more challenging question. There are many examples of four item costs (again each a positive-integer number of cents) with the sum equal to the product. Which of these gives the largest sum? Which gives the smallest?

Speed department

Ermanno Signorelli wants to know the next term in the sequence beginning 1, 2, 3, 5, 7, 11, 15, 22.

Solutions

Y2019. The following solution is from Steven Alexander.

$1 = 20 − 19$	$18 = (2 + 0 × 1) × 9$	$70 = (9 − 2) × 10$
$2 = 20/(1 + 9)$	$19 = 2 × 0 + 19$	$71 = 91 − 20$
$3 = 2 + 10 − 9$	$20 = 2^0 + 19$	$72 = 9 × (10 − 2)$
$4 = 9 − (10/2)$	$21 = 2 + 0 + 19$	$78 = 90 − 12$
$5 = (9 + 1 + 0)/2$	$22 = 21 + 9^0$	$80 = 9^2 + 0 − 1$
$6 = 9 − 1 − 0 − 2$	$27 = (2 + 0 + 1) × 9$	$81 = 9^2 × 1 + 0$
$7 = 9 + 1 × 0 − 2$	$28 = 20 − 1 + 9$	$82 = 92 − 10$
$8 = 9 + 1 + 0 − 2$	$29 = 20 × 1 + 9$	$87 = 90 − 1 − 2$
$9 = 2 × 0 + 1 × 9$	$30 = 20 + 1 + 9$	$88 = 90 − 1 × 2$
$10 = 20 − 1 − 9$	$38 = 2 × (0 + 19)$	$89 = 91 + 0 − 2$
$11 = 20 × 1 − 9$	$39 = 20 + 19$	$90 = 90 × (2 − 1)$
$12 = 20 + 1 − 9$	$44 = 90/2 − 1$	$91 = 91 + 0 × 2$
$13 = 12 + 9^0$	$45 = 9 × 10/2$	$92 = 92 + 0 × 1$
$14 = 9 + 5/1$	$46 = 90/2 + 1$	$93 = 92 + 0 + 1$
$16 = 2 × (0 − 1 + 9)$	$64 = (9 − 1 + 0)^2$	$95 = 190/2$
$17 = 9 + 10 − 2$	$69 = 90 − 21$	$100 = (9 + 1 + 0)^2$

S/O1. Mark Astolfi is interested in a "blockade" variation of chess stalemate in which the side to move has no moves at all, not even a move that would be illegal because it is moving into check. Among such positions he favors those with the fewest total number of pieces (including pawns).

The following solution from (my former NEC Research colleague) Lance Fortnow uses only six White pieces and the lone Black king. It is White's turn to move.
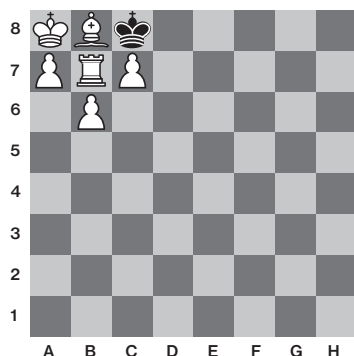
S/O2. Ermanno Signorelli has four specially marked dice. Their faces are 1-1-1-5-5-5, 2-2-2-2-6-6, 3-3-3-3-3-3, and 4-4-4-4-0-0. You first select one die and then your opponent selects one of the remaining three dice. Finally, each of you rolls your die and the higher number wins. Which die should you choose to maximize your chances of winning? Consider both an opponent who chooses among the remaining dice randomly and one who does a complete analysis. Greg Muldowney shows that with a "random opponent," you have an advantage going first, but this is not the case with an "analytical opponent."

Designating the dice in the order above as A, B, C, and D respectively, you can calculate the probability of your win on a single roll for all possible choices of dice:

Probability of win by your die			Your die					
			A		B		C	D
	Value		1	5	2	6	3	4	0
		Prob	1/2	1/2	2/3	1/3	1	2/3	1/3
Opponent's die	A	1				1/3	1/6	1/2	1/3	
		5	1/2				1/6			
	B	2	2/3		1/3			2/3	4/9	
		6	1/3							
	C	3	1		1/2		1/3		2/3	
	D	4	2/3		1/3		2/9			
		0	1/3	1/6	1/6	2/9	1/9	1/3		

From this, the overall probabilities of your winning are:

Prob of Win		Your die			
		A	B	C	D
Opponent's die	A		2/3	1/2	1/3
	B	1/3		2/3	4/9
	C	1/2	1/3		2/3
	D	2/3	5/9	1/3	
Mean		1/2	14/27	1/2	13/27

Thus you gain a very slight advantage by selecting B if your opponent chooses randomly among the remaining dice. If on the other hand your opponent does a full analysis, s/he has the upper hand regardless of your choice since s/he can always choose a die that reduces your odds of winning to 1/3. In this case you are actually disadvantaged by choosing first.

S/O3. I am embarrassed to report that problem S/O3 is a repeat of 2018 M/A3, so readers can find the solution in the 2018 J/A issue of the magazine.

Recall that essentially all back issues of the column as well as overflow material can be found on the Puzzle Corner website: https://cs.nyu.edu/~gottlieb/tr.

Better late than never

2019 M/A3. Carl Gordon notes that the deuce rules as stated are not quite those of tennis. The tennis rules have the server switch after every *multipoint* game, not every point, and sometimes tie-breakers are used.

2019 M/A1. The original author notes that I misinterpreted the problem, as explained in J/F1 above. In addition, Kells writes: "Even with the interpretation used, wouldn't the 33-point pattern K, AKQJ, AKQJ, AKQJ be a solution? (As an aside, the opponents may be able to make a grand slam against this hand and, if you 'sacrifice' at 7 no-trump, put you down 8!)" The 33-point pattern was also found by Bob Wake and J. Larsen.

Other responders

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Solution to speed problem

30. The *n*th term is the number of partitions of *n*.