Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

I am pleased to report that my beautiful wife, Alice, is now at the Mt. Sinai Medical System in NYC. Specifically, she is a clinical professor of dermatology at the Icahn School of Medicine and the medical director of the Department of Dermatology, Union Square. A side effect of the move is that we now have a small apartment in Manhattan in addition to our house in the suburbs.

Let me also add my condolences on the recent death of John Mattill. Among his many accomplishments, John brought Puzzle Corner to Technology Review and was a guiding light during the early years. (Originally, I'd been asked to create Puzzle Corner for the student publication Tech Engineering News; the column appeared in both places from 1966 until Tech Engineering News stopped publishing in 1976.) He will be missed by many, including me.

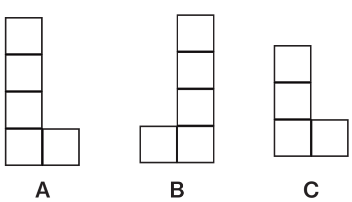
Problems

N/D 1. Our "games" problem this month, from Duncan Sheldon, is actually a combinatorics problem relevant to arranging an evening of bridge.

A bridge club consists of eight men and eight women who play at four tables. If the players are assigned so that all possible arrangements are equally likely, which of the following arrangements is more likely?

1. All partners are opposite sex.
2. All partners are same sex.

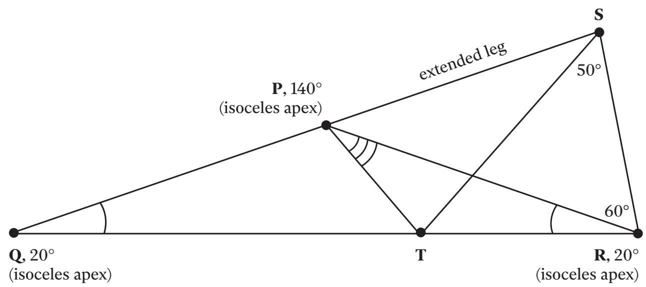
N/D 2. Nob Yoshigahara offers the following tiling problem from Osho in Kyoto. You are to construct an 8 x 8 square from four copies of A, four copies of B, and six copies of C. You may rotate the shapes but cannot flip them over.



N/D 3. Bruce Heflinger sent in this geometry problem:

In the diagram that follows, PQR is a 140-20-20 isosceles triangle, with leg QP extended as shown. A line segment is constructed from point R, 60° clockwise from leg RP, to meet the extension of QP at point S. A second line segment is constructed from point S, 50° clockwise from SR, intersecting QR at point T. A third line segment is constructed from point T to point P.

Evaluate angle TPR (marked with three nested arcs).

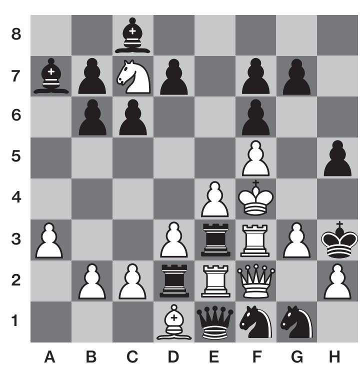


Speed department

Sorab Vatcha knows three siblings, the eldest of whom is 21 years old. The eldest notes that his age equals the sum of the other two ages and that the sum of two of the ages is twice that of the third age. What are the three (integer) ages?

Solutions

J/A 1. Another "Whose turn is it?" chess problem from Timothy Chow. Can the diagrammed position be reached by a sequence of legal moves from the standard starting position? If so, can you determine whose turn it is to move?



The following solution (it is White's move) is from Todd Chase, who proceeds via a process of elimination.

White's king was the last piece to enter the logjam on the first through fourth ranks, since all others in that sector are blocked by it. And they all must've entered the logjam before the e7xf6 capture, or the White king couldn't have made it to f4. So Black's h8 rook needed to escape via h6 earlier, eliminating h5 as the last move.

Since the White king went to f4 at the same time Black's f8 bishop was freed by the e7xf6 capture, and since the bishop still needed to get to a7, the White king couldn't have made the final move. The only remaining possibilities are White's knight, White's pawn on a3, or Black's dark-squared bishop.

Since the White king is a sitting duck against any Bishop checks on the h2-b8 diagonal, the White knight must have blocked checks for two successive moves as the bishop moved through c7 and b8 to a7. The only way this could've happened was if White played a2-a3 as the bishop moved to b8, so the pawn move is eliminated.

With only the knight and bishop as candidates, the only way to reach the final position is:

Nd6	Bb8
a3	Ba7
Ne8 (or b5)	Bb8+
Nc7	Ba7

So, in the diagrammed position, it must be White's move.

Chase gives a 40-move sequence to show how the diagrammed position is reached. His full solution is on the Puzzle Corner site.

J/A 2. Stephen R. Shalom has a large family and strange (to me) greeting customs. He reports that 18 members of his extended family came to visit, arriving two at a time. Their ages were 18 different integers; no one was over 100. As each pair arrived, Stephen computed the difference of the squares of their ages, and all nine times he got the same result. What were the 18 ages?

It seems everyone (or their computers) did some searching. Richard Lipes restates the problem as finding nine pairs (y, x) satisfying $0 < y, x \leq 100$ where $y^2 - x^2 = p = m \times n$ is constant. If we choose $y = (m + n)/2$ and $x = |m - n|/2$, a solution requires factoring $p = m \times n$ in nine ways with y and x either both even or both odd.

This can be done with $p = 1{,}440 = 2^5 \times 3^2 \times 5$ as follows.

$m = 5 \times 2^4$	$n = 3^2 \times 2^1$ (y, x)	$= (49, 31)$
$m = 5 \times 2^3$	$n = 3^2 \times 2^2$ (y, x)	$= (38, 2)$
$m = 5 \times 2^2$	$n = 3^2 \times 2^3$ (y, x)	$= (46, 26)$
$m = 5 \times 2^1$	$n = 3^2 \times 2^4$ (y, x)	$= (77, 67)$
$m = 5 \times 3 \times 2^3$	$n = 3^2 \times 2^2$ (y, x)	$= (66, 54)$
$m = 5 \times 3 \times 2^2$	$n = 3 \times 2^3$ (y, x)	$= (42, 18)$
$m = 5 \times 3 \times 2^1$	$n = 3 \times 2^4$ (y, x)	$= (39, 9)$
$m = 5 \times 3^2 \times 2^2$	$n = 2^3$ (y, x)	$= (94, 86)$
$m = 5 \times 3^2 \times 2^1$	$n = 2^4$ (y, x)	$= (53, 37)$

Several readers noted that $p = 2{,}880$ also "works," but two of the relatives have the same age (82).

J/A 3. I received a number of fine solutions to Robert Bird's geometry problem. Timothy Maloney solved it "while flat on my back and imagining diagonal lines tangent to a large 1/4 beach ball in the corner." The following solution (presumably done while seated) is from Zhe Lu.

We can express a point P on the P_3P_5 line as:

$P = a(P_3 - P_5) + P_5 = (-xa, ya, 2a) + (x, 0, -1) = ((1 - a)x, ya, 2a - 1)$ for some scalar value a. P_4 must be of this form.

Since P_4 lies on the unit sphere, we have:

$|P4|^2 = (1 - a)^2x^2 + y^2a^2 + 4a^2 - 4a + 1 = 1$

Rearranging to group by terms of powers of a:

$x^2 - 2ax^2 + a^2x^2 + y^2a^2 + 4a^2 - 4a = 0$

$(4 + x^2 + y^2)a^2 + (-2x^2 - 4)a + x^2 = 0$

This equation is quadratic in a. If P_3P_5 is to be tangent to the unit sphere, there must be exactly one real value of a that satisfies the above solution. Therefore, the discriminant must be zero:

$4x^4 + 16x^2 + 16 - 4(4 + x^2 + y^2)x^2 = 0$

$y = \pm\sqrt{4/x^2}$

As expected, there is a symmetry between x and y, since we can reverse the labels in the original question.

An extended solution, by Burgess Rhodes, is on the Puzzle Corner website.

Better late than never

J/A SD. Oh boy! Some sloppy wording on my part has led to a barrage of comments. In addition to questions of ferries and visiting a state three times, there is the embarrassing US-centric omission of starting and ending in Maine by driving through Canada upon reaching Washington. I should, and do, know better.

Other responders

Responses have also been received from A. Andersson, J. Bock, M. Branicky, J. Bross, D. Cane, L. Cangahuala, E. Cangahuala, G. Chan, J. Chandler, N. Derby, D. Dewan, A. Egendorf, D. Ertas, S. Feldman, M. Fichtenbaum, R. Frothingham, J. Glaser, M. Graetz, J. Harmse, C. Hesschen, W. Humann, T. Kerr, N. Lang, J. Langer, J. Larsen, W. Lemnios, S. Lerman, Z. Levine, D. Linden, R. Lipes, J. Mackro, P. Manglis, J. Marr, J. Martin, D. McIlroy, D. Mellinger, D.Micheletti, T. Mita, J. Mohr, R. Morgan, G. Muldowney, J. Murad, E. Passow, E. Person, P. Rakita, B. Rhodes, E. Signorelli, G. Steele, M. Stone, M. Strauss, S. Ulens, C. Wampler, J. Warshawsky, A. Wasserman, D. Weinberg, J. Weisman, and L. Zacks.

Solution to speed problem

7, v14, and 21 years.