Puzzle corner

Oh my gosh, we did it again! For the second time, we have referred to our very-much-alive contributor as "the late Dick Hess." This time, however, we understand the confusion: there are two Richard Hesses. Alumnus Richard L. Hess '57 died in 2016. Our Dick Hess is Richard I. Hess, who's been puzzling with us since he subscribed to Tech Review some 45 years ago.
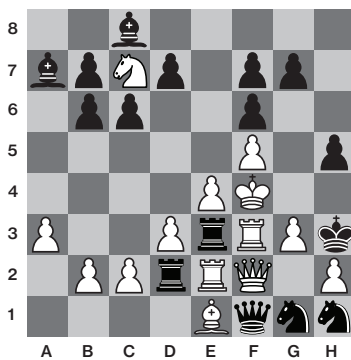
Game-related problems (chess, bridge, etc.) are in critical supply.

Problems

M/A 1. Another "Whose turn is it?" chess problem from Timothy Chow. Can the diagrammed position be reached by a sequence of legal moves from the standard starting position in chess? If so, can you determine whose turn it is to move?

M/A 2. Nob Yashigahara wants you to place the digits 1 through 9 once each into the nine boxes to yield a valid equation.

$$\frac{\square}{\square\,\square} + \frac{\square}{\square\,\square} + \frac{\square}{\square\,\square} = 1$$

M/A 3. In a number of two-person games, such as tennis or table tennis, a game can reach a "deuce" state in which the score is tied and the game proceeds as follows. Players alternate serving, and each serve leads to a point for one of the players. When one is up by two points, that player wins the game.

Assume player A, when serving, wins the point with probability P_A and player B, when serving, wins the point with probability P_B.

What is the probability that player A wins the game assuming A serves first at deuce? Is it better to serve first at deuce or second?

Speed department

Ermanno Signorelli spoke to a woman who said that she was 28 two days ago and will be 31 next year. How?

Solutions

N/D 1. Larry Kells wants you to find a bridge hand in which North-South can make some contract against best defense, even though neither of them has a card higher than a 9.

Jarek Langer apparently is well familiar with bad hands. He writes that South can make one spade on the layout below. If the lead is a minor suit, ruff in hand, and then cross-ruff three hearts and three more minor suit cards (two of the minor that wasn't led and one of the minor that was), ending in hand. If the lead is ace of hearts, let it win (do not ruff) and then play as above. East cannot over-ruff and pull trump before South has seven ruffing tricks.

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                  ♠ 9 8 7
                  ♥
                  ♦ 6 5 4 3 2
                  ♣ 6 5 4 3 2
  ♠                               ♠ A K Q J 10
  ♥ A                             ♥ K Q J 10
  ♦ A K Q J 10 9                  ♦ 8 7
  ♣ A K Q J 10 9                  ♣ 8 7
                  ♠ 6 5 4 3 2
                  ♥ 9 8 7 6 5 4 3 2
                  ♦
                  ♣
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N/D 2. Dave Blackston tells us that number theorists call a positive integer p-smooth if all its prime factors are at most p. The very large integer $N = 6^{100,000}$ is clearly 3-smooth. Blackston asks for the smallest 3-smooth number that is larger than N.

I received a number of wonderful solutions to this problem, but space concerns prevent me from including more of them. I wasn't going to select any that used floating-point arithmetic until I encountered one, by Tony Bielecki, that actually analyzed the rounding error. This solution is on the Puzzle Corner website.

Also on the website is Richard Bumby's solution, which includes a "continued fraction"–like analysis.

John Chandler explains easily why N/D 2 is just a (feasible) search problem (assuming your searching apparatus includes unbounded integers). Chandler writes:

"3-smooth integers are all of the form $2^A \times 3^B$, where A and B are nonnegative integers. The given number N is just the instance where $A = B = 100,000$. Obviously, if we start with a number that is greater than or equal to N and then decrease A or B by 1, we must increase the other exponent by at most 2 to get a new number that is again greater than N. It is therefore necessary only to examine all 200,000 pairs of A and B that give numbers minimally larger than N to see which is the smallest. The answer is $A = 225,743$, $B = 20,665$, yielding the value $e^{179,175.9469265}$ (compared with $N = e^{179,175.9469228}$)."

Jorgen Harmse put all his cards on the table and included his program, which is on the website. He writes: "The solution must be $2^k \times 3^l$, where k, l are nonnegative integers and $(k - 100,000)/(100,000 - l)$ is close to $\ln(3)/\ln(2)$. I don't have Matlab (which does rational approximations), but Python has unlimited integers, and a brute-force search takes a minute. I found $2^{225,743} \times 3^{20,665}$, which exceeds $6^{100,000}$ by about four parts in a million."

N/D 3. John Astolfi's "old-time safari puzzle" posits that in 1888 famed explorer Sir Rigglesworth was stymied. He wished to traverse on foot a totally barren desert that would take a person six days to cross. But a person could only carry only four days' rations of food and water. Fortunately, two of his bearers, Al and Zack, put their heads together and came up with a plan to get Sir Rigglesworth successfully across. How did they do it?

There are ambiguities in this problem. First, are we permitted to leave Al and Zack without supplies in the middle of the desert? I decided that we are not. Are we permitted to have the bearers start at the destination? Again, I voted no. Must all three get to the destination or may the bearers end at the start? This time I decided to permit both interpretations.

Jessica Winter-Stolzman lets the bearers return to base and gets Sir R. across in six days. All three start out together, each carrying four days' rations. After day 1, Al turns around, carrying only the ration she needs to get home, and gives her extra two days' worth to Zack and Sir R. After day 2, Zack also retreats with the two days' rations, and gives his extra to Rigglesworth. Now he has four days' worth of rations and four days of desert left to cross: success.

Rik Anderson gets all three across, but Sir R. needs 10 days.

Days 1–4: Al and Zack make two two-day round trips to Camp 1, caching two days' rations each on each trip; Sir R. joins them on one of the trips, so the cache has 10 days' rations total. Days 5–6: All three go from base to camp 2, each picking up one cached ration at camp 1, leaving seven there and arriving at camp 2 with three days' rations each, nine total. Day 7: Sir R. sets out to cross the desert with four days' rations, arriving across on day 10; Al and Zack return to camp 1 with one day's rations each, leaving three days' worth at camp 2. Day 8: Al and Zack return to camp 2 with five days' rations, which together with the three previously

cached there give the two of them enough to cross the remaining distance on days 9–12, arriving two days after Rigglesworth.

By comparison, David Goldfarb shows that without the bearers, Sir Rigglesworth can cross the desert in 12 days, as follows.

Day 1: Load up on the full four days' rations. Walk out into the desert. Day 2: Cache two days' rations; use the one day's worth left to return. Day 3: Load up on four days' rations again. Go to the previous cache. Day 4: Take one day's rations from the cache, and go one day further into the desert. Day 5: Cache two days' rations, then return to the first cache. Day 6: Return to camp. Days 7–8: Load up full again; go to the second cache. Days 8–12: Sir Rigglesworth consumed two days' rations getting to the cache. He refills there and is back up to four days' worth, with four days' worth of desert left to cross.

Better late than never

2018 J/A 1. Timothy Chow notes that many mate-in-two chess problems have a large number of variations and refers interested readers to https://goo.gl/GGtxqZ.

The key is unique (1.Qh5), but there are a lot of variations. After a "random" move by Black, any of the 13 moves by the bishop on d5 delivers checkmate, so we get well over a hundred variations this way, not to mention all the other variations where Black defends intelligently.

Other responders

Responses have also been received from R. Anderson, A. Andersson, K. Aou, M. Astolfi, F. Bachner, M. Barr, J.Bergmann, S. Berkenblit, E. Berlin, T. Bielecki, M. Bolotin, M. Branicky, M. Brill, R. Bumby, A. Cann, S. Carpenter, M. Chackerian, G. Chan, J. Chandler, T. Chase, T. Chow, G. Coram, R. Cross, B. Currier, R. DeJong, S. de Reil, D. Emmes, K. Feigl, G. Fischer, M. Galatin, D. Garbin, G. Garmaise, R. Giovanniello, D. Goldfarb, S. Gourley, D. Grunberg, A. Guthmiller, J. Hardis, J. Harmse, A. Hirshberg, J. Jamison, H. Joseph, U. Kausch, B. Kulp, J. Langer, J. Larsen, V. Levy, T. Mattick, J. McGrew, J. Mermelstein, B. Meyer, T. Mita, E. Moass, R. Morgen, G. Muldowney, S. Nelson, E. Nelson-Melby, S. Olgiati, A. Ornstein, R. Orr, S. Peters, B. Rhodes, K. Rosato, J. Russell, E. Signorelli, S. Silberberg, S. Silverstein, E. Smolisky, B. Snyder, S. Sperry, S. Ulens, R. Virgile, J. Ward, D. Worley, K. Zeger, and M. Zuker.

Solution to speed problem

She was born on December 31 and was speaking on January 1.